Testing Einstein’s Equivalence Principle With Gravitational Waves

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A conservative constraint on the Einstein Equivalence Principle (EEP) can be obtained under the assumption that the observed time delay between correlated particles from astronomical sources is dominated by the gravitational fields through which they move. Current limits on the EEP are mainly based on the observed time delays of photons with different energies, and it is highly desirable to develop more accurate tests involving different types of particles. The detection by the advanced LIGO/VIRGO systems of gravitational waves (GWs) will provide attractive candidates for constraining the EEP, which would further extend the tested particle species to the gravitons, with potentially higher accuracy. Considering the capabilities of the advanced LIGO/VIRGO network and the source direction uncertainty, we show that the joint detection of GWs and electromagnetic signals can potentially probe the EEP to an accuracy of $10^{-11}$, which is several orders of magnitude tighter than previous limits.

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I. INTRODUCTION

Albert Einstein’s Equivalence Principle (EEP) is one of the main cornerstones of general relativity as well as of many other gravitational theories. It states that any two different species of massless (or negligible rest mass) neutral particles, or two particles of same species with different energies, if emitted simultaneously from the same source and traveling through the same gravitational fields, should reach us at the same time \( \gamma_1 \). By measuring how closely in time the two different particles arrive, one can test the accuracy of the EEP through the Shapiro (gravitational) time delay effect. In practice, the EEP validity can be characterized by limits on the differences of the parametrized post-Newtonian (PPN) parameters (i.e., the parameter \( \gamma \), which denotes how much space curvature is provided by unit rest mass of the objects along or near the path of the particles) for different test particles (see, e.g., Refs. [4, 5]), since all metric theories of gravity incorporating the EEP predict \( \gamma_1 = \gamma_2 \equiv \gamma \), where the subscripts represent two different particles.

Any possible violation of the EEP would have far-reaching consequences for mankind’s view of nature, so it is important to extend the tests of its validity by making use of the panoply of new types of astronomical signals being brought to the fore in the multi-messenger era. So far, tests of the EEP through the relative differential variations of the \( \gamma \) values have been made using the emissions from supernova 1987A \( \gamma \), gamma-ray bursts (GRBs) \( \gamma \), fast radio bursts (FRBs) \( \gamma \), and TeV blazars \( \gamma \). Particularly, assuming as a lower limit that the observed time delay between different frequency photons from FRBs are caused mainly by the gravitational potential variations of the large scale structure, rather than the Milky Way’s gravity. It is important to extend the tests of its validity by making use of the panoply of new types of astronomical signals being brought to the fore in the multi-messenger era. So far, tests of the EEP through the relative differential variations of the \( \gamma \) values have been made using the emissions from supernova 1987A \( \gamma \), gamma-ray bursts (GRBs) \( \gamma \), fast radio bursts (FRBs) \( \gamma \), and TeV blazars \( \gamma \). Particularly, assuming as a lower limit that the observed time delay between different frequency photons from FRBs are caused mainly by the gravitational potential variations of the large scale structure, rather than the Milky Way’s gravity.
different energies. The first and only EEP tests with different species of particles are the measurements of the time delay of the photons and neutrinos from supernova 1987A [4, 5], where it was shown that the $\gamma$ values of photon and neutrino are equal to an accuracy of approximately 0.34%. New multi-messenger signals exploiting different emission channels are essential for testing the EEP to a higher accuracy. Recently, the Laser Interferometer Gravitational-wave Observatory (LIGO) team report their discovery of the first gravitational wave (GW) source, GW 150914 [12], opening a brand new channel for studying the Universe, which could lead to breakthroughs in both fundamental physics and astrophysics. In fact, the next generation of gravitational detectors, including the advanced LIGO, advanced VIRGO and KAGRA, appear poised to detect a plethora of increasingly sophisticated gravitational wave (GW) signals in the very near future [13–17]. In the following, we illustrate the possible progress in testing the EEP with the reported/future GW observations.

II. DESCRIPTION OF THE METHOD

The Shapiro time delay effect [2] causes the time interval for particles to pass through a given distance to be longer in the presence of a gravitational potential $U(r)$ by

$$\Delta t_{\text{gra}} = \frac{1 + \gamma}{c^3} \int_{r_e}^{r_o} U(r)dr,$$

(1)

where $\gamma$ is a PPN parameter, $r_o$ and $r_e$ correspond to locations of observation and the source of particle emission.

Assuming that the observed time delays ($\Delta t_{\text{obs}}$) between correlated particles from the same astronomical source are mainly caused by the gravitational potential of the Milky Way, and adopting the Keplerian potential for the Milky Way, we have [2, 12]

$$\Delta t_{\text{obs}} > \Delta t_{\text{gra}} = \frac{\Delta \gamma}{c^3} \frac{GM_{\text{MW}}}{c^3} \times$$

$$\ln \left( \frac{d + (d^2 - b^2)^{1/2}}{b^2} \right) \left[ r_G + s_n \left( \frac{r_G^2 - b^2}{2} \right)^{1/2} \right],$$

(2)

where $\Delta \gamma$ is the difference between the $\gamma$ values for different test particles, $M_{\text{MW}} \approx 6 \times 10^{11} M_\odot$ is the Milky Way mass [19, 21], $d$ represents the distance from the source to the center of Milky Way (if the source is of extra-galactic or cosmological origin, $d$ is approximated as the distance from the source to the Earth), $r_G \approx 8$ kpc is the distance from the Sun to the center of Milky Way, $b$ denotes the impact parameter of the particle paths relative to the Milky Way center, and $s_n = \pm 1$ is the sign of the correction of the source direction. If the source is located along the direction of Galactic center, $s_n = +1$.

While, $s_n = -1$ corresponds to the source located along the direction of anti-Galactic center. Note that the impact parameter $b$ is on the order of the distance of the Sun from the Galactic center, i.e., $b \leq r_G$. With Equation 2 one can constrain the EEP by putting a strict limit on the differences of $\gamma$ value [4–9].

We notice that although the method adopted in this work can provide severe constraints on the accuracy of the EEP, which is one of the important postulates of GR, it cannot be directly used to distinguish between specific gravity theories, such as GR and its alternatives. Many precise methods have been devised to test the accuracy of GR through the measurement of the absolute value of $\gamma$ based on the fact that GR predicts $\gamma = 1$. However, it is worth pointing out that $\gamma = 1$ is not a sufficient condition to identify general relativity, since 1) it is not the only theory that predicts $\gamma = 1$; 2) invalidation of the EEP or other postulates of GR might cause the absolute $\gamma$ value in GR to deviate from unity [2]. Thus, further investigations would be essential for developing more accurate tests of the EEP and for distinguishing between GR and other alternative gravity theories.

III. EEP TEST USING GW SIGNALS

The process of compact binary coalescence (CBC; either neutron star (NS) binary, black hole (BH) binary or NS-BH binary) provides the primary targets for the second generation of GW detectors, such as the advanced LIGO/VIRGO [13–17]. The first reported GW detection, GW 150914, is a BH-BH merger with two BH masses $36_{-14}^{+5} M_\odot$ and $29_{-13}^{+5} M_\odot$, respectively [12]. Of significant interest for CBC GW detections is the fact that some relevant fundamental physics postulates, including the EEP, may be constrained using gravitational radiation alone [2, 30]. This could be done exploiting the fact
that the frequency of the gravitational radiation sweeps from low frequencies at the initial moment of observation (in-spiral phase) to a higher frequency at the final moment (coalescence phase), or sweeps from higher frequency where the signal amplitude reaches a maximum to lower frequency (ring-down phase). Any EEP violation will cause a distortion of the observed phasing of the waves, and would result in a shorter (or longer) than expected overall time of passage of a given number of cycles. Provided that the parameters of the compact binary can be determined accurately, it will be essential to compare the theoretical template phase with the observations for obtaining severe constraints on the accuracy of the EEP. In this case, since no EM counterparts are required, all CBC GW detections would be relevant.

Phenomenologically, one can treat the different frequency GWs as different gravitons to test the EEP. For instance, the signal of GW 150914 increases in frequency and amplitude in about 8 cycles (over 0.2 s) from 35 to 150 Hz, where the amplitude reaches a maximum [12]. Considering the localization information of GW 150914, we could tighten the limit on the EEP to $\Delta \gamma \sim 10^{-9}$. Moreover, the detected GW signals emitted from the ringdown phase of the newly formed stellar mass BH swept the detector band (e.g., 100 Hz to 200 Hz) within a time of order of milliseconds, which could further improve the constraint on the EEP to $\Delta \gamma \sim 10^{-11}$.

More recently, the Fermi GBM team reported that GBM observations at the time of GW150914 reveal the presence of a weak transient source above 50 keV, 0.4 s after the GW event was detected, with a false alarm probability of 0.0022 [21]. If this is indeed the EM counterpart of GW150914, as supported by the theoretical interpretations [22, 23], with the aforementioned method, we could further extend the EEP test with gravitons and photons, setting a severe limit on EEP to an accuracy of $10^{-5}$, five orders of magnitude tighter than the results set by the photons and neutrinos from supernova 1987A.

Besides BH-BH mergers, GW signals from binary NSs and NS-BH mergers are also expected to be detected in the near future, for which a variety of detectable electromagnetic (EM) counterparts have been widely discussed [24, 26], including the following representative cases: the prompt short GRB emission, the afterglow emission of the on-beam ultra-relativistic outflows, and the macronova/kilonova emission of the sub-relativistic r-process material ejected during the merger. For NS-NS mergers, if the merger product is a massive millisecond pulsar instead of a BH, the detectable EM signatures from the system become much richer and brighter (see Ref. [27, 28] for details). Joint detections of GW/EM signals, once achieved, could be used to give important constraints on the EEP.

Consider the case of a joint detection of GW/EM signals from a NS-NS or NS-BH coalescence event in the advanced LIGO/VIRGO era. Since the sky and binary orientation averaged sensitivity of the advanced LIGO/VIRGO network for CBC is of the order of $\sim 100$ Mpc [13, 17], here we assume the distance from the GW source to the Earth to be $d = 200$ Mpc. It is worth pointing out that the constraints on the EEP are not greatly affected by the source distance uncertainty (see Ref. [8] for more explanations). To account for the source direction uncertainty, and based on the fact that the impact parameter $b \leq r_G$, here we present four extreme cases by assuming $a = 0.001r_G$ and $s_n = +1$, $b = 0.001r_G$ and $s_n = -1$, $b = 0.999r_G$ and $s_n = +1$, and $b = 0.999r_G$ and $s_n = -1$, respectively. The real results should lie within the range circumscribed by these extreme cases.

Regarding the EM counterpart of the GW detection, suppose we are lucky to detect all the promising emission types, e.g. the short GRB prompt emission, the on-beam GRB afterglow emission and the macronova emission. Recently, Ref. [29] discussed the time lags between the GW signal and all these EM counterparts in some detail, and suggested that the time delay $\Delta t_{\text{obs}}$ is expected to be of the order of $\sim 0.01$–1 s (short GRB), 0.01–1 day (on-beam afterglow), or 1–10 days (macronova), respectively. With these expected time delays and with the location information in hand, we would be able to set bounds on the EEP from Equation (2). The expected constraints on the differences of the $\gamma$ values are shown in Figure 1. It has been suggested that the macronova emission may be the most frequently-detectable EM signal of the coalescence events [25, 26]. If the macronova emission is detected at $\Delta t_{\text{obs}} \sim 1$ day after the merger, a strict limit on the EEP will be $\Delta \gamma < 10^{-5}$. One can see from this plot that much more severe constraints would be achieved ($\sim 10^{-3}$–$10^{-5}$ or $10^{-8}$–$10^{-10}$) if the EM counterpart is an on-beam afterglow or a short GRB. Note that the compact binary coalescence and the EM counterpart do not occur at the same time, since $\Delta t_{\text{lag}}$ has a contribution from the intrinsic emission time lag ($\Delta t_{\text{lag}}$) between the photons and the GW signals. Here we take $\Delta t_{\text{lag}} = 0$ to give a conservative estimate of the EEP. More severe constraints could be achieved with a better understanding of the nature of $\Delta t_{\text{lag}}$ allowing one to remove its contribution from $\Delta t_{\text{obs}}$. On the other hand, it should be underlined that these upper limits are based on very conservative estimates of the gravitational potential of the Milky Way. If the gravitational potential fluctuations from the intervening large scale structures are taken into considered, our constraint results would be further improved by orders of magnitude [10, 11].

IV. SUMMARY AND DISCUSSION

In conclusion, we show that new EEP tests can be carried out with potentially much higher accuracy in the upcoming GW era. For all kinds of CBC GW detections, regardless of whether EM counterparts are detected or not, we can always use different frequency gravitons to give severe constraints on the accuracy of the EEP. Taking GW 150914 as an example, it takes a few milliseconds for the GW signals emitted from the ringdown phase of
the newly formed stellar mass BH to sweep the detector band (e.g. 100Hz to 200Hz), resulting in a tightening of the limit on the EEP to approximately $10^{-11}$, which is three orders of magnitude better than the current most stringent results from FRBs. 8.

Once EM counterparts of the GW signal are firmly detected, interesting EEP test could be performed by using the measured time delay between the gravitons and any associated photons. Also taking GW 150914 as an example, if the claimed short GRB, GW150914-GBM, is indeed the EM counterpart of GW150914, a severe limit on EEP could be set to an accuracy of $10^{-8}$, five orders of magnitude tighter than the results set by the photons and neutrinos from supernova 1987A 4, 5.

Finally, considering the capabilities of the advanced LIGO/VIRGO network and the source direction uncertainty, we found that for the expected GW detection from NS-NS/BH mergers, if the prompt short GRB emission and/or its afterglow emission is detected, a stringent limit on the EEP could be set at the level of $\Delta \gamma < (10^{-8} - 10^{-10})$ (prompt) or $\sim 10^{-3} - 10^{-5}$ (afterglow). Due to the low detection rates of GRB- accompanied GW signals, the first positively identified electromagnetic counterpart of a GW signal is very likely to be a macronova. If the macronova emission is detected at $\Delta t_{\text{obs}} \sim 1$ day after the merger, a strict limit on the EEP will be $\Delta \gamma < 10^{-3}$.

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[22] Metzger, B. D., Berger, E. 2012. What is the Most


