A Tracker Solution for a Holographic Dark Energy Model

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Abstract

We investigate a kind of holographic dark energy model with the future event horizon the IR cutoff and the equation of state -1. In this model, the constraint on the equation of state automatically specifies an interaction between matter and dark energy. With this interaction included, an accelerating expansion is obtained as well as the transition from deceleration to acceleration. It is found that there exists a stable tracker solution for the numerical parameter $d > 1$, and d smaller than one will not lead to a physical solution. This model provides another possible phenomenological framework to alleviate the cosmological coincidence problem in the context of holographic dark energy. Some properties of the evolution which are relevant to cosmological parameters are also discussed.

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Numerous and complementary cosmological observations lend strong support to present acceleration of our universe [1]. Some negative-pressure source is needed to meet these observational requirements. Since its energy density is unexpected small ($\rho_\Lambda \sim 10^{-47} GeV^4$) in the framework of QFT, to understand this amazing phenomenon is a great challenge to the fundamental physics [2]. Despite a variety of phenomenological dynamical fields (quintessence, phantom, k-essence, etc) with suitable chosen potentials [3], the simplest and most confusing candidate $-$ a cosmological constant (CC) Λ $-$ is still attractive. As a matter of fact, this is a long-term topic and several proposals have been discussed in the past twenty years, such as the screening of CC due to Hawking radiation or the existence of an infrared fixed point in effective theories of gravitation [4], the relaxation of CC based on the coincidence limit of the graviton propagator growing in time [5] and the screening mechanism due to virtual gravitons [6]. Nevertheless, part of these mechanisms have some ability to be compatible with subsequent observations of type Ia Supernovae which indicate present cosmic acceleration, although most of them were primarily focused on inflationary cosmology. The theoretical enchantment of Λ and recent remarkable observational evidence on the equation of state (EOS) w_{DE} around -1 [2, 7] makes theorists try to reconcile a dynamical dark energy (DE) with a true cosmological constant $-$ hereafter we mean $w_{DE} = -1$. The congruence of these two aspects has been stimulated these days. In Ref. [8] a kind of back-reaction-induced cancellation mechanism is trying to understand the seemingly varying CC. With the Hubble parameter H the renormalization scale, a picture of the running CC has been developed in Ref. [9]. And we note that some phenomenological approaches to study the decaying vacuum cosmology [10, 11] have also emerged.

On the other hand, the holographic principle [12] has been inspiring great endeavors on attack of this fine-tuning quantity ρ_Λ. In a seminal paper of Cohen [13], there is a suggestion that in QFT a short distance cutoff is related to a long distance cutoff due to the limit set by formation of a blackhole, and this ever neglected IR limitation to QFT will correspond to an energy scale of $10^{-2.5} eV$ if the IR cutoff is our present Hubble scale $H_0^{-1} \approx 10^{28} cm$. According to Refs. [13]-[15], this choice of IR cutoff will alleviate the cosmological constant problem [2]. In line with this suggestion, Hsu [16] and Li [17] argued that this energy density could be viewed as the holographic DE density satisfying $\rho_{DE} = 3d^2 M_P^2 L^{-2}$, where M_P is the reduced Planck mass. Li also demonstrated that only identifying L with the radius of the future event horizon R_e, we can get the EOS $w_{DE} < -1/3$ and an accelerating universe. Generally speaking, the universe was in the matter dominated era until recently. As a consequence, today's particle horizon and Hubble horizon are roughly of the same order [19]. Then for Li's model to share the merit of Hubble scale cutoff on reproducing the correct magnitude of cosmological constant, the following question emerges: why is the future event horizon R_e comparable to the particle horizon and the Hubble horizon at present? It may be regarded as a holographic variation of cosmological coincidence problem (CCP), which concerns about the mysterious approximate equality of matter and DE density today [18].

Those suggestions on holographic DE have been extensively discussed in the past few

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years [15, 20, 21, 22]. It was found that after considering the interaction between dark matter and DE, the choice of H^{-1} as the IR cutoff of the holographic DE may be compatible with the desired $w_{DE} = -1$ [15]. Nevertheless, with that interaction included, the scaling behavior of the CC which is in favor of the CCP may also be obtained. After loosing the constraint on $w_{DE} = -1$, Ref. [21] has also realized the acceleration and a scaling solution. And there the transition from deceleration to acceleration requires a varying coefficient d. Lately, a model which uses R_e instead of H^{-1} as IR cutoff and leaves w_{DE} undetermined has also recovered the acceleration transition and moreover an EOS transition from $w_{DE} > -1$ to $w_{DE} < -1$ phantom regimes [22] (see also [23]).

It can be seen that, studies up to now have shown the scaling solution in the Hubble horizon criterion of holographic DE, no matter whether the EOS is fixed to be -1. However, the future event horizon cutoff which is used to scale the holographic DE clearly leads to a finale of $\Omega_{DE} = 1$ [17], where Ω_{DE} is the DE fraction of the total energy density. This is unable to address the CCP. In this Letter, we study a particular holographic DE model [24] and exhibit the theoretical possibility to drive an R_e version of holographic DE to understand the CCP. The basic ingredients are as follows: the holographic DE scales as R_e^{-2}, but different from previous models it is assumed to possess a constant EOS $w_\Lambda = -1$, i.e., we here deal with a varying but "true" CC.

The holographic DE density is

$$\rho_\Lambda \equiv 3d^2 M_p^2 R_e^{-2}, \tag{1}$$

here we keep d as a free positive dimensionless parameter and R_e is the proper size of the future event horizon,

$$R_e(t) \equiv a(t) \int_t^\infty \frac{d\tilde{t}}{\tilde{a}(\tilde{t})} = a \int_a^\infty \frac{d\tilde{a}}{\tilde{H}\tilde{a}^2}, \tag{2}$$

where a is the scale factor of the universe. For a spatially flat, isotropic and homogeneous universe with an ordinary matter and dark energy, the Friedmann equation can be written as

$$\Omega_\Lambda + \Omega_m = 1, \quad \Omega_m \equiv \frac{\rho_m}{\rho_{cr}} \quad \text{and} \quad \Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_{cr}}, \tag{3}$$

where $\rho_m(\rho_\Lambda)$ is the energy density of matter (dark energy) and the critical density $\rho_{cr} = 3M_p^2 H^2$. Because of the conservation of the energy-momentum tensor, the evolution of the energy of matter and DE are governed by

$$\begin{aligned}\dot{\rho}_\Lambda &= Q \\ \dot{\rho}_m + 3H\rho_m &= -Q\end{aligned} \tag{4}$$

respectively. Here we have used the requirement

$$w_\Lambda = -1 \tag{5}$$

and Q represents the undetermined interaction between matter and DE.

By definition Eq. (1), we have

$$R_e^2 \equiv \frac{3d^2 M_p^2}{\rho_\Lambda} = \frac{d^2}{\Omega_\Lambda H^2} . \tag{6}$$

According to the definition of the future event horizon (2), a straightforward calculation can give

$$\dot{R}_e = HR_e - 1. \tag{7}$$

Hereafter the superscript dot denotes the derivative with respect to the cosmic time t. Then the rate of change of both energy components may be expressed as

$$\dot{\rho}_\Lambda = 6M_P^2 H^3 \Omega_\Lambda \left(\frac{\sqrt{\Omega_\Lambda}}{d} - 1 \right), \tag{8}$$

and

$$\dot{\rho}_m = -6M_P^2 H^3 \Omega_\Lambda \left(\frac{\sqrt{\Omega_\Lambda}}{d} - 1 \right) - 9M_P^2 H^3 (1 - \Omega_\Lambda) \tag{9}$$

where Eq. (3) has been recalled. It's clear that the energy density of matter and DE can not be conservative respectively in our model. There is energy transfer between those two energy components and the coupling term Q is just of the form $H\rho_\Lambda$ multiplied by a variable coefficient. In the present framework of our model, the truly independent continuity equation is Eq. (9) and it may be employed to produce the evolution equation for Ω_Λ. By means of Eq. (3), Eq. (9) can be cast into

$$\Omega_\Lambda' = -3\Omega_\Lambda^2 + \frac{2}{d}\Omega_\Lambda\sqrt{\Omega_\Lambda} + \Omega_\Lambda, \tag{10}$$

where the prime denotes the derivative with respect to $x \equiv \ln a$ and then $\dot{\Omega}_\Lambda = \Omega_\Lambda' H$. To study the scaling behavior of the cosmological evolution, it's convenient to introduce an auxiliary quantity [27]

$$r \equiv \frac{\rho_m}{\rho_\Lambda} = \frac{1 - \Omega_\Lambda}{\Omega_\Lambda}. \tag{11}$$

The rate $\dot{r}$ of the energy density ratio r of matter and dark energy can be written as

$$\dot{r} = \left(\frac{\rho_m}{\rho_\Lambda} \right)^{\cdot} = -\frac{\dot{\Omega}_\Lambda}{\Omega_\Lambda^2}. \tag{12}$$

Then Eq. (10) becomes

$$\dot{r} = \frac{H(3d\Omega_\Lambda - 2\sqrt{\Omega_\Lambda} - d)}{d\Omega_\Lambda} = 0. \tag{13}$$

where $\dot{r} = 0$ gives the possible cosmological scaling behavior $\sqrt{\Omega_\Lambda^+} = (1 + \sqrt{1 + 3d^2})/(3d)$. When the parameter d is greater than 1, the positive root $\sqrt{\Omega_\Lambda^+}$ is smaller than 1 and then a meaningful scaling solution and vice versa. Moreover, the larger d is, the smaller value

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such a scaling solution can take. When the DE component fraction is smaller than the physical scaling solution Ω_Λ^+, then $\dot{r} < 0$ and $\Omega_\Lambda' > 0$ and as a result the DE fraction will monotonically increase up to the maximum Ω_Λ^+. In practice, this is most relevant to our universe since the DE component is expected to play a more and more role in the evolution with the flow of cosmological time. As a consequence, the observational data of present DE component fraction Ω_Λ^0 will give an observational upper bound of $d \leq 2\sqrt{\Omega_\Lambda^0}/(3\Omega_\Lambda^0 - 1)$ by way of the $\Omega_\Lambda^0 \leq \Omega_\Lambda^+$. For example, if we choose $\Omega_\Lambda^0 = 0.7$, then the parameter d should not be larger than 1.5. People might worry about whether the parameter d can take such values greater than 1 since the original bound $L^3\rho_\Lambda \leq LM_P^2$ proposed by Cohen [13] will be violated. Careful analysis suggests this may not be the case. The model we suggest is only a phenomenological framework and it's unclear whether it's appropriate to tightly constrain the value of d by means of the analogue to the blackhole physics. As a matter of fact, the possibility of $d > 1$ has been seriously dealt with and a modest value of d larger than one could be favored in the literature [25]. When d equals to 1, the positive root $\Omega_\Lambda^+ = 1$ and the cosmic expansion approaches a de Sitter phase asymptotically. How about the value $d < 1$? Unlike the original holographic dark energy model [17, 20] with R_e the IR cutoff where the universe approaches a phantom phase for $d < 1$, numerical simulations indicates that there exists no consistent physical solution. In fact, $d < 1$ always makes Ω_Λ' positive and $\dot{r} < 0$, even though the increasing DE fraction has reached 1 with matter component vanishing. This would then ensue from an unacceptable negative matter density. We should note that, some previous fits to the observational SN Ia data [20, 22] in the context of holographic dark energy were on the basis different from our model and the best fits which suggest a free parameter d smaller than 1 may be irrelevant to present model.

In order to study the stability of the critical point of Eq. (10) which corresponds to the scaling solution Ω_Λ^+, substituting a linear perturbation $\Omega_\Lambda \to \Omega_\Lambda^+ + \delta$ about the critical point into the Eq. (10), to first-order in the perturbation, gives

$$\delta' = \left(-6\Omega_\Lambda^+ + \frac{3}{d}\sqrt{\Omega_\Lambda^+} + 1\right)\delta. \tag{14}$$

It is easy to check that the scaling solution Ω_Λ^+ is always the late-time stable attractor solution.

Now let's turn to more details of this model which are relevant to some other observational quantities. The transition of deceleration to acceleration happened when

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_P^2}(\rho_\Lambda + 3p_\Lambda + \rho_m) = 0. \tag{15}$$

Using Eqs. (3) and (5) as well as the above formula we can obtain that the transition emerges at $\Omega_\Lambda^T = 1/3$, which is irrelevant to the parameter d. This result coincides with that of the standard LCDM scenario since both models have the same EOS of minus one.

The deceleration parameter q is

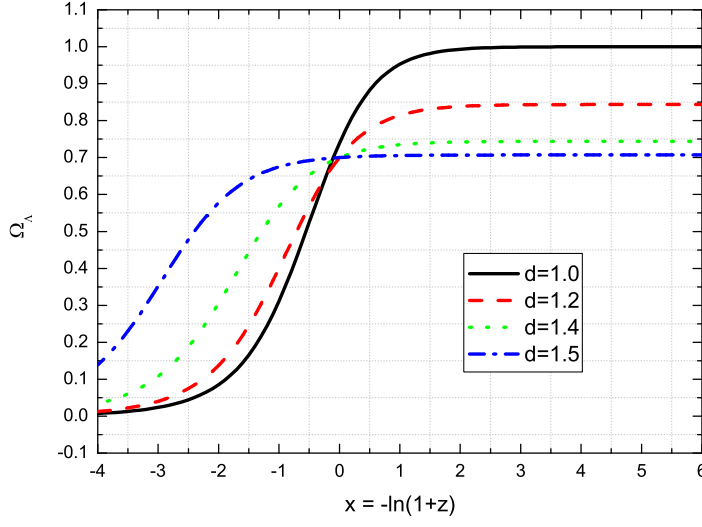$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{H'}{H} - 1. \tag{16}$$

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Figure 1: Evolution of the DE for different values of the constant d.

Through Eqs. (2) and (6), we have

$$\frac{d}{\sqrt{\Omega_\Lambda}Ha} = \int_a^\infty \frac{d\tilde{a}}{\tilde{H}\tilde{a}^2} = \int_x^\infty \frac{d\tilde{x}}{\tilde{a}} \tag{17}$$

and taking derivative with respect to $\tilde{x}$ in both sides of the above equation we get

$$\frac{H'}{H} = \frac{\sqrt{\Omega_\Lambda}}{d} - \frac{\Omega'_\Lambda}{2\Omega_\Lambda} - 1. \tag{18}$$

Using the information extracted from Eq. (10), the deceleration parameter q may be determined by virtue of Eqs. (16) and (18). If exhibit the evolution of this model in redshift, we may note that $1 + z = 1/a$ (by conventional we have chosen present scale factor $a_0 = 1$) and then there is a relation $x = -\ln(1 + z)$.

After specifying the value of the parameter d, the behavior of the DE evolution may be obtained through Eq. (10). The dependence of the evolution of DE with respect to the constant d is shown in Fig. 1. We see that for different values of the parameter $d > 1$, the evolution of the universe will approach different tracker solutions. The solution Ω_Λ^+ is illustrated as the plateau of a particular curve in Fig. 1. What's more, the larger d is, the more gently Ω_Λ climb up to an end value and the less such a value. For $d = 1$, the evolution will approach a de Sitter universe and the energy component of matter will be infinitely diluted. Once again, we should note that, in our model $d < 1$ would yield an unphysical solution and then is not allowed.
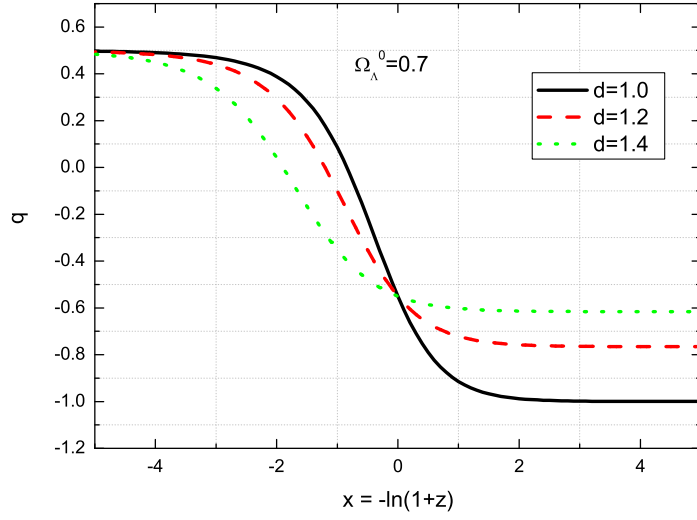
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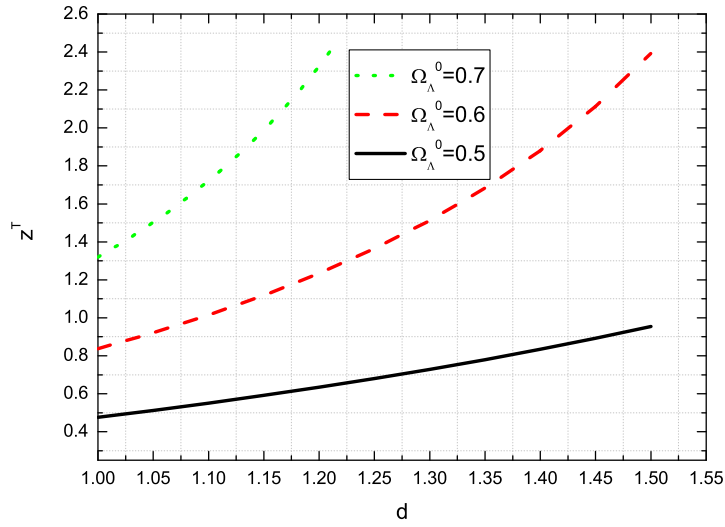Figure 2: Dependence of the deceleration parameter on the constant d.

Figure 3: Dependence of the deceleration parameter on the constant d.

Fig. 2 shows the evolution of the deceleration parameter q where for definiteness the value of Ω_Λ^0 is set to be 0.7. The discussion above have shown that in this case $d \in [1, 1.5)$. From this figure, we can easily see that $q_0 = -0.5$ is independent of the parameter d and there is a transition from deceleration to accelerating expansion. Fig. 3 shows the relation between the redshift of the turning point z_T at which the deceleration expansion to acceleration transition happened and the value of the parameter d. It's obvious that the evaluation of present DE density fraction imposes a practical constraint on the parameter d which is indicated with the rapidly ascending curve.

In conclusion, we have studied a kind of holographic DE model in which the future event horizon is chosen to be the IR cutoff and the equation of state is fixed to be -1. In this model, an interaction between matter and dark energy naturally appears. We find that the accelerating expansion as well as the transition from deceleration to acceleration is well recovered. There exists a stable tracker solution for the dimensionless parameter $d > 1$. So this model provides one possible phenomenological framework to alleviate the cosmological coincidence problem with the holographic motivation. We show that, by means of only one cosmological parameter the DE density fraction, the constant d obtains an observational upper bound. Specifically speaking, if today's universe is DE dominated, there is a practical constraint on the numerical factor d which can not deviate much from 1. It's interesting to further examine this model with current observational data and determine whether other strategies such as a varying Newton's constant [9, 15, 24, 28] are necessary to extend our framework while making the cosmological coincidence problem still ameliorated.

Acknowledgements

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