Nuclear Science and Techniques **24** (2013) 050505

Density isomer of nuclear matter in an equivalent mass approach

GAO Li[1,2] LU Zhenyan[1] PENG Guangxiong[1,3,*]

[1]*College of Physics, University of Chinese Academy of Sciences, Beijing 100049, China*

[2]*Department of Physics, Shandong University of Technology, Zibo 255049, China*

[3]*Theoretical Physics Center for Science Facilities, Institute of High Energy Physics, Beijing 100049, China*

Abstract The equation of state of symmetric nuclear matter is studied with an equivalent mass model. The equivalent mass of a nucleon has been expanded to order 4 in density. We first determine the first-order expansion coefficient in the quantum hadron dynamics, then calculate the coefficients of the second to fourth order for the given binding energy and incompressibility at the normal nuclear saturation density. It is found that there appears a density isomeric state if the incompressibility is smaller than a critical value. The model dependence of the conclusion has also been checked by varying the first-order coefficient.

Key words Density isomer, Equivalent mass, Nuclear equation of state

1 Introduction

Modern nuclear physics has recently made much progress in understanding a number of interesting phenomena, such as the monotonic decreasing direct flow in the collective motion of relativistic heavy ion collisions[1], the extension of the symmetry energy from heavy-ion collisions at intermediate energies suggests a soft symmetry energy[2], the mass reduction of the pressure and transition temperature in single-flavor color superconductivity[3], an analytic expression of nuclear symmetry energy to the fourth order[4], the first direct observation of the deexcitation of the low-lying isomeric state $^{229\mathrm{m}}$Th from photon emission[5], etc.

Isomeric states are very interesting and important to the nuclear equation of state (EoS). It is a common sense that there is a minimum in the density dependence of the average energy per nucleon with the corresponding density normally called the nuclear saturation density. Early in 1950s, people had already realized that a second minimum, the so-called density isomer, could exist in nuclear matter[6]. The density isomer could arise from phase transition[6], pion condensation[7-10], the three-body force[11], etc. Normally, it is not very easy to produce a second minimum in nuclear EoS because it originates, most probably, from higher-order contributions. In principle, people have the fundamental theory of strong interactions, the Quantum Chromodynamics (QCD). However, due to the difficulty in the treatment of interaction in non-perturbative regime and the consistent implementation of chemical potential for finite density, phenomenological models have been popularly applied. On the other hand, a non-interaction system is exactly solvable. One can, therefore, use an equivalent particle mass to mimic the interaction effect. The equivalent mass approach has been extensively applied, e.g., in studying properties of quark matter[12-16], strangelets[17], in-medium chiral condensates[18-20], strange quark stars[21], and QCD phase transition[22].

As for an application to nuclear matter, we have previously seen that the equivalent-mass approach produces the nuclear saturation when the equivalent mass was expanded to a Taylor series to order three in density[20]. The purpose of the present paper is to check if the equivalent-mass model can produce the density isomer by extending the equivalent mass of nucleons to the fourth order. We first determine the first-order expansion coefficient in the quantum hadrondynamics, then calculate the

---

Supported by National Natural Science Foundation of China (NSFC) Projects (Nos.11135011 and 11045006) and the key project from Chinese Academy of Sciences (No.12A0A0012)

* Corresponding author. *E-mail address*: gxpeng@ucas.ac.cn

Received date: 2013-06-30

coefficients of the second to fourth order for the given binding energy and incompressibility at the normal nuclear saturation density. It is found that there exists a density isomeric state if the incompressibility is smaller than a critical value. The model dependence of the conclusion has also been checked by varying the first-order coefficient.

The paper is organized as follows. In the subsequent Section 2, we describe the equivalent-mass model suitable for symmetric nuclear matter, while the relevant thermodynamics treatments are presented in Section 3. Then numerical results and discussions are provided in Section 4. And finally a summary is given in Section 5.

2 Equivalent-mass model for symmetric nuclear matter

As mentioned in the introduction, the model we use tries to include interaction between particles in an equivalent mass. This equivalence is ensured by the proper variation of the particle mass with density. With the definition of the equivalent mass, the energy density of symmetric nuclear matter can be written as the same form with that of free particles, i.e.,

$$\varepsilon = \frac{g}{2\pi^2}\int_0^{\nu}\sqrt{p^2+M^2}\,p^2dp, \tag{1}$$

where g=2×2=4 is the degeneracy factor. M is a density-dependent nucleon mass to be determined by expansion a little later. The Fermi momentum ν is connected to the nucleon number density ρ by

$$\rho = \frac{g\nu^3}{6\pi^2} \quad \text{or} \quad \nu = \left(\frac{6\pi^2}{g}\rho\right)^{1/3}. \tag{2}$$

Explicitly, the integration in Eq.(1) can be carried out to give

$$\varepsilon(\nu,M) = \frac{g}{16\pi^2}\left[\nu(2\nu^2+M^2)\sqrt{\nu^2+M^2} - M^4\operatorname{arcsh}\left(\frac{\nu}{M}\right)\right]. \tag{3}$$

Also from Eq.(1), one can get

$$\frac{\partial\varepsilon}{\partial M} = \frac{gM}{2\pi^2}\int_0^{\nu}\frac{p^2dp}{\sqrt{p^2+M^2}}. \tag{4}$$

Or, one explicitly has the expression

$$\frac{\partial\varepsilon}{\partial M} = \frac{gM}{4\pi^2}\left[\nu\sqrt{\nu^2+M^2} - M^2\operatorname{arcsh}\left(\frac{\nu}{M}\right)\right]. \tag{5}$$

The second derivative can also be obtained as

$$\frac{\partial^2\varepsilon}{\partial M^2} = \frac{g}{4\pi^2}\left[\frac{\nu\left(\nu^2+3M^2\right)}{\sqrt{\nu^2+M^2}} - 3M^2\operatorname{arcsh}\left(\frac{\nu}{M}\right)\right]. \tag{6}$$

With a glance at Eq.(4), we immediately know that $\partial\varepsilon/\partial M>0$, i.e., ε is a monotonically increasing function of the nucleon mass at any fixed density, one can, therefore, give any value of ε with an equivalent density dependent nucleon mass, given the energy density is not smaller than

$$\varepsilon = \frac{g}{8\pi^2}\left(\frac{6\pi^2}{g}\rho\right)^{4/3}, \tag{7}$$

which is obtained by setting M=0 in Eq.(1), or taking the limit $M\to 0$ in Eq.(3).

Figure 1 demonstrates this in an obvious way, where the energy density is plotted, according to Eq.(3), as a function of the nucleon mass at different values of the nucleon number density. Generally it is a monotonically increasing function.

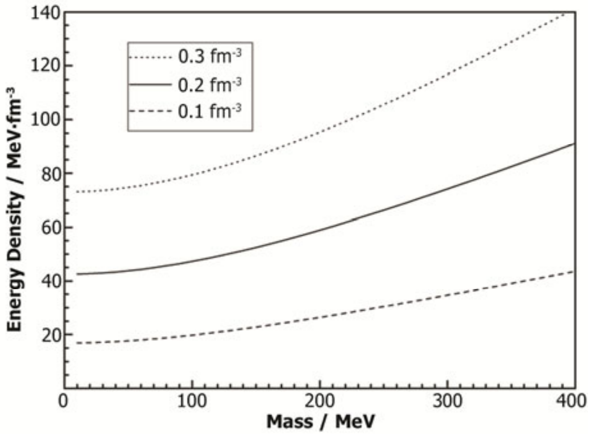

Fig.1 The energy density as a function of the nucleon mass at different fixed density. The three curves correspond to different fixed values of the density indicated in the legend.

Therefore, if one can get the energy density from some other models or even from QCD possibly in the future, we solve the equation $\varepsilon=E_{\text{mod}}$ for the equivalent mass.

In the covariant meson-baryon effective field theories of the nuclear many-body problem (often called quantum hadron dynamics or QHD[23,24]), for example, we have[20]

$$M = M_N - \frac{1}{2}\left(\frac{g_\sigma^2}{M_\sigma^2} - \frac{g_\omega^2}{M_\omega^2}\right)\rho, \tag{8}$$

where M_N=938.926 MeV is the average mass of nucleons in free space. M_σ and M_ω are, respectively, the masses for the sigma and omega mesons. g_σ and g_ω are the corresponding coupling constants. It is noticeable that the equivalent mass contains contributions not only from the scalar σ meson, but also from the vector ρ meson. This is different from the conventional effective masses that depend merely on the scalar meson.

The first derivative of the energy per nucleon with respect to the density satisfy

$$\frac{d}{d\rho}\left(\frac{\varepsilon}{\rho}\right) = -\frac{\varepsilon}{\rho^2} + \frac{1}{\rho}\left(\sqrt{\nu^2+M^2} + \frac{\partial\varepsilon}{\partial M}\frac{dM}{d\rho}\right), \tag{9}$$

where the expression of the partial derivative $\partial\varepsilon/\partial M$ has been given in Eq.(5). Similarly, for the second derivative we have

$$\begin{aligned}\rho\frac{d^2}{d^2\rho}\left(\frac{\varepsilon}{\rho}\right) &= \frac{2\varepsilon}{\rho^2} + \frac{2\pi^2}{g\nu\sqrt{\nu^2+M^2}} - \frac{2\sqrt{\nu^2+M^2}}{\rho} \\ &+ \left(\frac{2M}{\sqrt{\nu^2+M^2}} - \frac{2}{\rho}\frac{\partial\varepsilon}{\partial M}\right)\frac{dM}{d\rho} \\ &+ \frac{\partial^2\varepsilon}{\partial M^2}\left(\frac{dM}{d\rho}\right)^2 + \frac{\partial\varepsilon}{\partial M}\frac{d^2M}{d\rho^2}.\end{aligned} \tag{10}$$

3 Thermodynamic treatment

Special attention should be paid to the consistency of thermodynamic formulas, when the particle mass depends explicitly on density. At zero temperature, the main expressions are the energy density and the pressure. In literature, there are three kinds of treatments. In the first treatment, both the energy density and pressure expressions have the same form with those of the constant-mass case[12]. In the second one, both the energy density and pressure have an additional term due to the density dependence of the particle mass[13]. Finally in the third treatment, the additional term are added only to the pressure, but not to the energy density[14,15,22]. These treatments were originally provided for quark matter with density-dependent quark masses. In the present context of nuclear matter, the energy density and pressure expressions for the first treatment are

$$E_1 = \varepsilon,\ P_1 = -\Omega_0; \tag{11}$$

for the second treatment, these become

$$\begin{aligned} E_2 &= \varepsilon - \rho\frac{dM}{d\rho}\frac{\partial\varepsilon}{\partial M}, \\ P_2 &= -\Omega_0 + \rho\frac{dM}{d\rho}\frac{\partial\varepsilon}{\partial M}; \end{aligned} \tag{12}$$

for the third treatment, they are

$$E_3 = \varepsilon,\ P_3 = -\Omega_0 + \rho\frac{dM}{d\rho}\frac{\partial\varepsilon}{\partial M}. \tag{13}$$

In Eqs.(11-13), the ε expression is in Eq.(1) or Eq.(3), while the quantity Ω_0 is the same form as the thermodynamic potential density of a free system, but with the constant particle mass replaced by a density-dependent one, i.e.,

$$\Omega_0 = \varepsilon - \rho\sqrt{\nu^2+M^2}. \tag{14}$$

Replacing the p with υ in the squared root on the right hand side of Eq.(1), one immediately finds that $\varepsilon < \rho\sqrt{\nu^2+M^2}$. This leads inevitably to P_1>0, i.e., the pressure in the first treatment can never be less than zero. This means the equation of state is always a monotonic line, which is obviously not correct. In the second treatment, the pressure can be zero or negative. However, the zero pressure is not consistent with the minimum of the energy per nucleon. A little later we will see that the pressure should be exactly zero at the energy minimum. One can check, the third treatment satisfies thermodynamic consistency requirement.

In fact, the chemical potential can be obtained from $d\varepsilon/d\rho$, giving

$$\mu = \sqrt{\nu^2+M^2} + \frac{dM}{d\rho}\frac{\partial\varepsilon}{\partial M}. \tag{15}$$

In normal thermodynamic formulas, one has only the first term on the right. In the present case, we have an additional term, i.e., the second term in Eq.(15) which occurs due to the density dependence of the nucleon mass. It also appears in the pressure

$$\begin{aligned} P &= -\varepsilon + \mu\rho \\ &= -\varepsilon + \rho\sqrt{\nu^2+M^2} + \rho\frac{dM}{d\rho}\frac{\partial\varepsilon}{\partial M}. \end{aligned} \tag{16}$$

The extra term is very important to ensure thermodynamic consistency[22]. From Eqs.(9), (15), and (16), we can easily check that

$$P = \rho^2 \frac{d}{d\rho}\left(\frac{\varepsilon}{\rho}\right). \tag{17}$$

This expression can also be directly derived from the fundamental differential equality:

$$d(V\varepsilon) = -PdV + \mu dN, \tag{18}$$

where V is the volume. N is the particle number. ε is the energy density. $V\varepsilon$ is the system energy. Eq.(18) is nothing but a combination of the first and second laws of thermodynamics at zero temperature. Because ρ=N/V, simple derivation from Eq.(18) gives

$$\begin{aligned} P &= -\left.\frac{d(V\varepsilon)}{dV}\right|_N = -\frac{d}{dV}\left(\frac{N\varepsilon}{\rho}\right)_N = -N\frac{d}{dV}\left(\frac{\varepsilon}{\rho}\right)_N \\ &= -N\frac{d\rho}{dV}\frac{d}{d\rho}\left(\frac{\varepsilon}{\rho}\right)_N = \frac{N^2}{V^2}\frac{d}{d\rho}\left(\frac{\varepsilon}{\rho}\right), \end{aligned} \tag{19}$$

which leads to Eq.(17) directly. Please note the subscript N means that the corresponding derivative is taken at a fixed N.

Eq.(17) explicitly shows that the pressure is exactly zero at an energy extreme (minimum or maximum) where the first derivative of the energy per nucleon with respect to density vanishes.

If one would like to express the thermodynamic quantities with the effective chemical potential,

$$\mu^* \equiv \sqrt{\nu^2 + M^2}, \tag{20}$$

then Eqs.(14), (15), and (16) become

$$\begin{aligned} \Omega_0 = -\frac{g}{48\pi^2}\Big[& \mu^*(2\mu^{*2} - 5M^2)\sqrt{\mu^{*2} - M^2} \\ & +3M^4 \ln\frac{\mu^* + \sqrt{\mu^{*2} - M^2}}{M}\Big], \end{aligned} \tag{21}$$

$$\mu = \mu^* + \frac{dM}{d\rho}\frac{\partial\Omega_0}{\partial M}, \tag{22}$$

$$P = -\Omega_0 + \rho\frac{\mathrm{d}M}{\mathrm{d}\rho}\frac{\partial\Omega_0}{\partial M}. \tag{23}$$

Omitting the effective indicator (the superscript $*$) and the free system indicator (the subscript 0) will give the previously applied corresponding formulas[14-16]. Therefore, the chemical potential there should be understood as an effective one. Because the effective and real chemical potentials differ only by a common quantity, both of them satisfy the same weak-equilibrium equations. Consequently, the numerical calculations in using only the effective chemical potential are all right. However, if one wants to study phase diagram, the real chemical potential must be calculated in the model. This point was explicitly pointed out in Ref.[22].

It should be mentioned that a recent paper[25] criticized the third treatment, and finally returned back to the first treatment. As shown in the above, however, the first treatment can not give any negative pressure to maintain mechanical equilibrium, and the pressure is never zero at the energy minimum (if any). Secondly, the paper did not distinguish between the effective and real chemical potentials, and the thermodynamic inconsistency is inevitable (if using their formulas to do calculations in phase transition, one should prepare to meet unexpected results). Thirdly, and more seriously, it modified the fundamental differential relation of thermodynamics by introducing “an intrinsic degree of freedom m^*” which depends obviously and explicitly on density. In this regard we would comment that the authors should modify their model to obey the fundamental theory of thermodynamics, rather than doing the inverse. We therefore continue to apply the third treatment in the present calculations. For a fully-consistent thermodynamic derivation and explanation, both at zero and finite temperate, one may refer to Ref.[22].

4 Numerical results and discussion

The density dependence of the equivalent nucleon mass is in principle very complicated. A natural requirement is

$$\lim_{\rho\to 0} M(\rho) = M_{\mathrm{N}}, \tag{24}$$

therefore, it can be expanded to a series in density as

$$M = M_N \sum_{i=0}^{\infty} a_i \left(\frac{\rho}{\rho_{\mathrm{sc}}}\right)^i, \tag{25}$$

where the quantity ρ_{sc} is a scale parameter to let the expansion coefficients a_i be dimensionless. In principle, its value can be arbitrarily chosen. Naturally, different choice of ρ_{sc} will lead to different values of a_i. However, the choice does not change the actual physics, and the concrete values of a_i do not change with choosing different unit systems. In the present calculation, we take the natural value of $\rho_{\rm sc}$= 0.17 fm^{-3}, which is approximately the nuclear situation density.

In principle, there should be an infinite number of terms in Eq.(25). We now consider the expansion to order 4, i.e.,

$$\frac{M}{M_N}=a_0+a_1\frac{\rho}{\rho_{\rm sc}}+a_2\left(\frac{\rho}{\rho_{\rm sc}}\right)^2 +a_3\left(\frac{\rho}{\rho_{\rm sc}}\right)^3+a_4\left(\frac{\rho}{\rho_{\rm sc}}\right)^4. \qquad (26)$$

According to Eq.(24), we naturally have a_0=1. Comparing Eq.(25) with Eq.(8), we get

$$a_1=-\frac{\rho_{\rm sc}}{2M_N}\left(\frac{g_\sigma^2}{M_\sigma^2}-\frac{g_\omega^2}{M_\omega^2}\right). \qquad (27)$$

The relevant meson masses and couplings are taken to be g_σ=10.6, M_σ= 545 MeV, g_ω=12.6, M_ω= 782 MeV. These values give a_1= –0.082545.

For higher order coefficients, there are no direct expressions to be compared with. However, at the saturation density ρ_0, we have

$$\left[\frac{\varepsilon}{\rho}\right]_{\rho=\rho_0}=M_N+E_b, \qquad (28)$$

$$\left[\frac{d}{d\rho}\frac{\varepsilon}{\rho}\right]_{\rho=\rho_0}=0, \qquad (29)$$

$$\left[9\rho^2\frac{{\rm d}^2}{{\rm d}\rho^2}\frac{\varepsilon}{\rho}\right]_{\rho=\rho_0}=K, \qquad (30)$$

where $\rho_0\approx$ 0.17 fm^{-3} is the normal nuclear saturation density. $E_b\approx$ –15.9 MeV is the nuclear binding energy at the saturation. These quantities and the nucleon mass $M_{\rm N}$ are very nicely fixed. For a given value of the corresponding incompressibility K, we can determine the coefficients a_2, a_3, a_4. For example, for K=300 MeV, 200 MeV and 160 MeV, the corresponding a_2, a_3 and a_4 are listed in Table 1.

Table 1 Expansion coefficients at order-4 level. The binding energy is taken to be $E_{\rm b}\approx$ –15.9 MeV at the saturation density $\rho_0\approx$ 0.17 fm^{-3}

K/MeV	a_1	a_2	a_3	a_4
160	–0.082545	0.058185	–0.020297	0.0025215
200	–0.082545	0.060614	–0.025155	0.0049502
300	–0.082545	0.066686	–0.037298	0.0110219

In Fig.2, we show the binding energy per nucleon, $\varepsilon/\rho - M_N$, as a function of density for several typical values of the incompressibility. One can see that all the curves go through the saturation point which is a energy minimum. Because the pressure is zero and the curve is concave (the second derivative is positive), the energy minimum is mechanically stable.

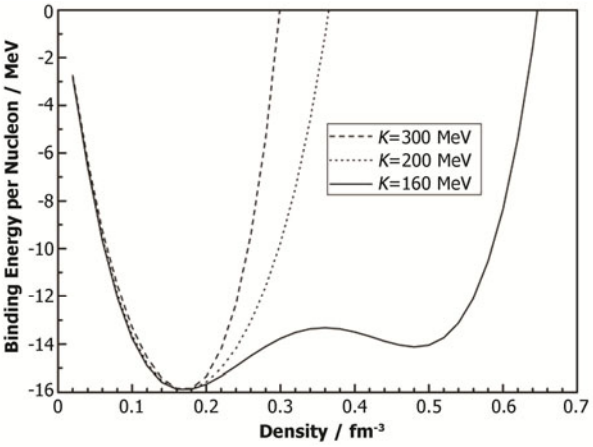

Fig.2 The binding energy per nucleon of symmetric nuclear mass as a function of density. All the curves saturate at $\rho_0\approx$ 0.17 fm^{-3} with the binding energy $E_b\approx$ –15.9 MeV. The corresponding incompressibility is indicated in the legend. Noticeably, if the incompressibility is small, there exists the second minimum.

There are two cases when the density goes higher. For bigger incompressibility, it increases monotonically with increasing density above the saturation. For the case of a smaller incompressibility, however, there appears another point where the pressure is also zero. However, the curve is convex (the second derivative is negative), it is thus a maximum. When the density goes further higher, we see another minimum which is an isomeric state, or density isomer.

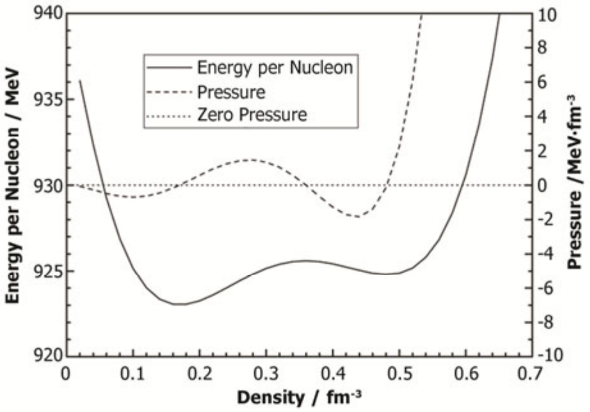

Fig.3 The density isomer in nuclear matter is a mechanically stable and physically meta-stable state. The energy of this density isomer is a little bit higher than the energy state at the saturation. The difference can be several KeV to several MeV depending on the parameters.

The density isomer appears at about three times the nuclear saturation density. In Fig.3, the energy per nucleon, ε/ρ, and the corresponding pressure is simultaneously shown. It is obvious that the pressure at the second minimum is zero. The isomer is also mechanically stable. Its energy per nucleon is merely a little bit higher than the energy per nucleon at the normal saturation density.

In the above calculation, we have given the value for the coefficient a_1 by Eq.(27) from the QHD. We now check how the value of a_1 influences the isomer because a different model can easily give a different a_1 value. For this we need to discuss the value of K.

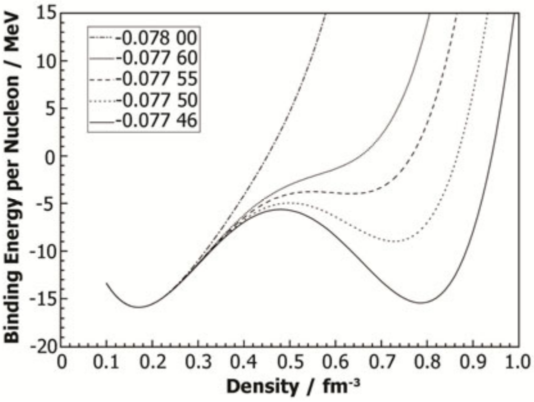

Fig.4 Variation of the density isomer on the order-1 expansion coefficient a_1 with values indicated in the legend. The density isomer exists only for properly smaller a_1 values.

The incompressibility is the curvature of the equation of state and thus measure the stiffness of nuclear matter at the saturation density. Its value is presently not completely determined though the iso-scalar giant monopole resonance provides a direct experimental tool to study nuclear incompressibility in finite nuclear systems[26]. The early calculation with relativistic models[27], where the contributions from negative energy sea had been included, gave values of 250–270 MeV. Recently, non-relativistic models[28,29] give a little bit smaller values of 220–233 MeV. The experimental determination comes from a study of the iso-scalar giant monopole resonance or the breathing mode state. In this regard a data fit based on a model of nuclear energy functional and the scaling assumption of the nuclear breathing mode gives (220±20) MeV[30]. We thus use $K = 220$ MeV to check the model dependence of the isomer. For the given incompressibility, we manually give an a_1, rather than calculate it from Eq.(27). For a_2, a_3, a_4, they can still be solved from Eqs.(28-30).

In Fig.4, we plot, at $K = 220$ MeV, the binding energy per nucleon as a function of the density for different chosen values of a_1. For smaller values of a_1, there exists the isomeric state. With increasing a_1, the energy difference between the two minima also increases. When a_1 increases to a critical value (– 0.07755 in Fig.4), the density isomer disappears.

5 Conclusion

We have studied the density isomer in the symmetric nuclear equation of state with the equivalent mass of nucleons expanded to order 4 in density. It is found that the density isomer exists if the nuclear incompressibility is comparatively smaller. The density isomer is the second energy minimum at about three times the normal nuclear saturation. Its average binding energy can be several KeV to MeV.

However, the concrete values should not be taken seriously because the present treatment is rather rough, and more investigations in details are needed. Application of the equivalent-mass model to the study of symmetry energy may be helpful, and it is suitable to be given elsewhere in the future.

References

1 Guo Y, Liu F, Tang A H. Phys Rev C, 2012, **86:** 044901.

2 Kumar S and Ma Y G. Phys Rev C, 2012, **86:** 051601.

3 Wu P P, Hou D F, Ren H C. Phys Rev D, 2012, **84:** 125031.

4 Cai B J, Chen L W. Phys Rev C, 2012, **85:** 024302.

5 Zhao X X, de Escobar Y N M, Rundberg R, *et al*. Phys Rev Lett, 2012, **109:** 160801.

6 Lee T D and Wick G C. Phys Rev D, 1974, **9:** 2291–2316.

7 Migdal A B. Rev Mod Phys, 1978, **50:** 107–172.

8 Brown G E and Weise W. Phys Rep, 1976, **27:** 1–34.

9 Migdal A B, Sorokin G A, Markin O A, *et al.* Phys Lett B, 1976, **65:** 423–426.

10 Hecking P and Weise W. Phys Rev C, 1979, **20:** 1074–1078.

11 Nyman E M and Rho M. Nucl Phys A, 1977, **290:** 493–500.

12 Chakrabarty S, Raha S, Sinha S. Phys Lett B, 1989, **229:** 112–116.

13 Benvenuto O G and Lugones G. Phys Rev D, 1995, **51:** 1989–1993.

14 Peng G X, Chiang H C, Zou B S, *et al*. Phys Rev C, 2000, **62:** 025801.

15 Wen X J, Zhong X H, Peng G X, *et al*. Phys Rev C, 2005, **72:** 015204.

16 Zhang Y and Su R K. Phys Rev C, 2002, **65:** 035202.

17 Wen X J, Peng G X, Chen Y D. J Phys G: Nucl Part Phys, 2007, **34:** 1697–1709.

18 Peng G X, Lombardo U, Loewe M, *et al*. Phys Lett B, 2002, **548:** 189–195.

19 Peng G X. Nucl Phys A, 2005, **747:** 75–83.

20 Peng G X, Chiang H Q, Ning P Z. Int J Mod Phys A, 2003, **18:** 3151–3174.

21 Zheng X P, Liu X W, Kang M, *et al*. Phys Rev C, 2004, **70:** 015803.

22 Peng G X, Li A, Lombarto U. Phys Rev C, 2008, **77:** 065807.

23 Serot B D and Walecka J D. Int J Mod Phys E, 1997, **6:** 515–631.

24 Serot B D and Walecka J D. Adv Nucl Phys, 1986, **16:** 1–321.

25 Yin Y and Su R K. Phys Rev C, 2008, **77:** 055204.

26 Khan E. Phys Scripta T, 2012, **150:** 014018.

27 Lalazissis G A, Konig J, Ring P. Phys Rev C, 1997, **55:** 540–543.

28 Piekarewicz J. Phys Rev C, 2004, **69:** 041301.

29 Soubbotin V E, Tselyaev V I, Vinas X. Phys Rev C, 2004, **69:** 064312.

30 Chung K C, Wang C S, Santiago A J. Phys Rev C, 1999, **59:** 714–721.