Directed flow of isospin sensitive fragments within a modified clusterization algorithm in heavy-ion collisions

KUMAR Sanjeev[1,2] MA Yugang[2,*]

[1]*Amity Institute of Nuclear Science and Technology, Amity University, Noida, Uttar Pradesh-201303, India*

[2]*Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Jiading Campus, Shanghai 201800, China*

Abstract By extending the minimum spanning tree （MST）clusterization algorithm for the binding energy cut， the isospin asymmetry dependence of directed flow for isospin sensitive isobar pairs (neutrons-protons, ^{3}H-^{3}He) is studied from low towards high incident energies. The modified clusterization method (MSTB) has the advantage to identify the fragments at quite early time. It enhances (reduces) the production of free nucleons (fragments) over MST method. The directed flow of isobaric pair ^{3}H-^{3}He is more sensitive towards isospin asymmetry caused by MSTB than isobaric pair *n*-*p*. This sensitivity becomes quite strong towards the high incident energy and neutron-rich reaction system. In conclusion, the inclusion of binding energy in clusterization method for the flow studies has been uniquely important for understanding the isospin physics, especially for high density behavior of symmetry energy.

Key words Binding energy, Isospin Physics, Symmetry energy, Directed flow

1 Introduction

With the availability of radioactive beam facilities, isospin physics has attracted the whole of the nuclear community in last decade. In intermediate energy heavy-ion collisions, the dynamical models such as Quantum Molecular Dynamical (QMD)[1-15], Boltzmann-Uehling-Uhlenbeck (BUU)[16-22] and statistical models[23-27] were used to get the rich information about isospin physics. The dynamical models are of unique importance due to their ability to follow the time evolution from initialization through compression and expansion to equilibrium state. It is also well known that dynamical models do not simulate the fragments. In order to simulate the fragments, one needs the secondary algorithm for coupling with dynamical models. In the efforts to reproduce the experimental data, many secondary algorithms have been developed to study the nuclei near the drip line[11-13,15,28,29], but very few for nuclei away from drip line[5,6,8,10,15].

The most commonly and widely used algorithm depends on the spatial and momentum coordinates of the nucleons, is known as minimum spanning tree (MST) algorithm[12,15]. According to this method, two nucleons undergo the cluster formation if the relative distance ($|R_i–R_j|$) and relative momentum ($|P_i–P_j|$) between the nucleons was less than 3.5–4 fm and 250–268 MeV/*c*, respectively. These parameters can be obtained by fitting the experimental data for some of the global observables such as multiplicity of intermediate mass fragments[12,15] with theoretical results. Recently, MST method was further extended to isospin-dependent MST, in which the cut on the momentum space was kept same, but the cut on the spatial coordinates was constraint on the basis of type of particles. The distance between the different kind of particles was taken as follow: $|R_i^p - R_j^p| = 3$ fm, $|R_i^n - R_j^p| = |R_i^n - R_j^n| = 6$ fm[14].

Since the MST method only depends on the constraints from position and momentum, it seems to be worried about the stability of fragments due to the formation of artificial weakly bound fragments. To

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* Corresponding author. *E-mail address:* ygma@sinap.ac.cn

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avoid from this problem, more complicated methods like Stimulated Annealing Clusterization Algorithm (SACA)[13,28], Early Cluster Recognization Algorithm (ECRA)[29] were also developed. These methods were found quite successful, but, very complicated. Moreover, due to the choice of parameters such as cooling parameters, iterations procedure, and choice of minimal can follow the totally different configuration of fragments. Due to the limitation of sharp minimal, in the mildly excited or asymmetric systems, the scope of these two methods was found to restrict at some points. The possible best method for avoiding from the artificial formation of fragments is found a way to constrain the fragments by using an average binding energy cut of 4 MeV/nucleon[30]. One can further improve the method by using the realistic binding energy rather than 4 MeV/nucleon[30]. This method was found to be as simple as MST and found to reproduce the experimental data just like the complicated methods SACA and ECRA etc. The extensive studies for the multiplicity of different kind of fragments with different clusterization methods have been performed[12-15,28,30], while flow parameters are poorly known with this type of methods[1,8,11,12,31,32].

In last two decades, the collective flow of nucleons with non-central collisions was found as a useful tool for extracting the nuclear equation of state （NEOS） of symmetric nuclear matter[31,32]. With the passage of time, efforts have been done to extract the information of isospin physics in term of symmetry energy using the directed as well as elliptic flow[8,21,32]. The studies showed that the experimental results of directed flow can be reproduced by (1) changing the mean field or nucleon-nucleon （*NN*）cross sections in the transport model (2) using the nucleon phase space for calculations[31,33,34]. Most of the studies were found to obey the hard equation of state[33-35], which opposed the findings with other observables in the literature[12,19,28,31,32,36-38]. In recent study, the soft equation of state was predicted[39], but, the comparison was made between the directed flow extracted from phase space of nucleons and the directed flow of fragments having $Z \leqslant 2$[40]. Within the BUU calculations[21], the importance of directed flow of isospin sensitive fragments *n*, *p*, ^{3}H and ^{3}He over the directed flow of nucleon phase space has already been put forwarded. The elliptic flow extracted with the simple MST method was also found to be quite sensitive towards the determination of symmetry energy at supra-saturation densities[8]. The results indicated that the directed as well as elliptic flow can be a useful tool to understand the isospin physics or symmetry energy.

The main problem with the study was that isospin physics with directed and elliptic flow is either studied from the nucleon phase space or by using the simple MST method. It is worth mentioning here that one cannot guarantee about the stability of fragments formed with simple MST method. Due to the worry about the stability of fragments, it is the prime need of the present time to study the importance of Isospin physics from directed and elliptic flow with the stable fragments.

In the present study, first the stable fragments are formed by applying the binding energy cut on the pre-clusters formed with the simple MST method. Afterward, the sensitivity of the isospin sensitive fragments *n*, *p*, ^{3}H and ^{3}He directed flow is studied from low towards the high incident energy with the variation in isospin asymmetry of the reaction systems.

The modified clusterization method is explained in Section-2, the results and discussions are presented in Section-3, followed by the conclusion in Section-4.

2 Methodology

In the present work, IQMD model is used, which is discussed in detail in our recent publications[5,6,10], originally developed by Hartnack and Co-workers[11-13]. The model is modified by the authors for the density dependence of symmetry energy, having form:

$$E_{\rm Sym}(\rho)=\frac{C_{s,k}}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3}+\frac{C_{s,p}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

with the parameters of $C_{s,k}$ = 25 MeV and $C_{s,p}$ = 35.2 MeV. When we set γ_i =0.5 and 1.5, respectively, it corresponds to the soft and stiff symmetry energy[5,6].

Minimum Spanning Tree method with

Binding Energy Check （MSTB）is used. This method is a modified version of the normal MST and old MSTB method. The difference between old MSTB and this MSTB is the inclusion of energy from momentum dependent interactions as well as symmetry energy along with Skyrme interactions. The procedure is as follow: the phase space obtained from IQMD is analyzed with simple MST method and pre-clusters are sort out. Since we are not aware about the stability of pre-clusters formed at this stage, the pre-clusters formed from the simple MST are now subjected to the binding energy condition as follow:

$$\frac{1}{N_f}\sum_{\alpha=1}^{N_f}\left[\sqrt{(\mathbf{p}_\alpha-\mathbf{P}_{N_f})^2+m_\alpha^2}-m_\alpha+\frac{1}{2}\sum_{\beta\neq\alpha}^{N_f}V_{\alpha\beta}\right]<-E_{\mathrm{Bind}}.$$

Here, we take E_{Bind} = 4.0 MeV/nucleon if $N_f\geq3$ and E_{Bind} = 0 otherwise. In this equation, N_f is the number of nucleons in a fragment, $\mathbf{P}_{N_f}$ is the average momentum of the nucleons bound in the fragment. The requirement of a minimum binding energy excludes loosely bound fragments which will decay later. The realistic value of E_{Bind} changes slightly the fragment multiplicity at intermediate times, but has no influence on the qualitative behavior and on the asymptotic results. However, if by using the realistic binding energy, one searches for the most bound configuration, the results will get affected. At the present time, we had just focused on the bound configurations and hence the average cut of binding energy –4 MeV/nucleon is justified.

3 Results and discussions

Several thousands of event for the reactions of ^{112}Sn+^{112}Sn and ^{124}Sn+^{124}Sn between the incident energy 50 and 600 MeV/nucleon for the nearly central collisions using the IQMD model coupled with MST and MSTB algorithms are simulated. The soft momentum dependent equation of state with soft symmetry energy and isospin-energy dependent cross sections is employed.

There are two methods in the literature used to calculate the directed flow. In the first case, the directed flow is extracted from the mid-rapidity slope of the P_x/A versus rapidity distribution $Y_{\mathrm{c.m.}}/Y_{\mathrm{beam}}$ plots. The rapidity distribution is calculated as follow:

$$Y(i)=\frac{1}{2}\ln\frac{\mathbf{E}(i)+\mathbf{p}_z(i)}{\mathbf{E}(i)-\mathbf{p}_z(i)},$$

where $\mathbf{E}(i)$ and $\mathbf{p}_z(i)$ are the total energy and longitudinal momentum of ith particle, respectively.

The second method is to study the directed transverse in-plane flow P_x^{dir} is as follow:

$$\left\langle P_x^{\mathrm{dir}}\right\rangle=\frac{1}{A}\sum_i^A \mathrm{sign}\{Y(i)\}P_x(i),$$

where $Y(i)$ is the rapidity distribution as discussed above and $P_x(i)$ is the transverse momentum of the ith particle in x-direction. This P_x^{dir} is defined over entire rapidity region and therefore expected to present an easier way of measuring the in-plane flow rather than complicated P_x/A plots.

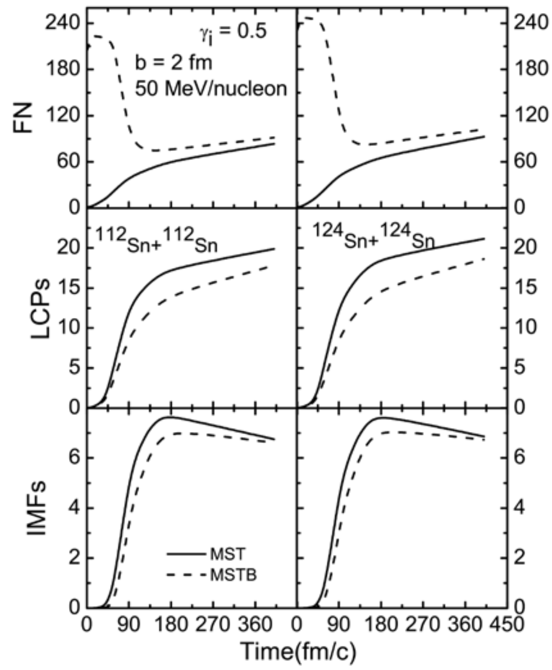

Fig.1 Time evolution of different kind of fragments: free nucleons (upper), LCPs (middle) and IMFs (bottom) for central collisions at 50 MeV/nucleon with MST and MSTB algorithms. The left (right) panels are for neutron-poor (neutron-rich) ^{112}Sn+^{112}Sn（^{124}Sn+^{124}Sn） reaction systems. All the results are with the soft symmetry energy.

In order to check the stability of fragments and effect of isospin physics, in Fig.1, the time evolution of free nucleons, light charged particles （LCPs）and intermediate mass fragments （IMFs） for neutron-poor and neutron-rich reaction systems at 50 MeV/nucleon with MST and MSTB algorithms is displayed. In the high density phase, the MSTB algorithm does not find any fragments with reasonable binding energy and hence most of the particles are free, while, there were a lot of artificial fragments with MST. After the high density phase is over, it starts recognizing the fragments, which are real, bound and stable. In all the cases, MSTB helps to identify the fragments quite early. From the figure, it is clear that MSTB enhances the production of free particles and reduces the production of LCPs and IMFs. Moreover, with increase in the size of the fragment, MST takes less time to match with the MSTB results and also the difference between MST and MSTB results goes on decreasing throughout the time evolution.

When the results are compared for neutron-poor and neutron-rich reaction systems, it is found that enhanced production of fragments takes place for the more neutron-rich system. The enhanced production is quite effective in free nucleons and LCPs compared to IMFs. It is also shown by us recently that free nucleons and LCPs are more sensitive towards the symmetry energy compared to IMFs[10]. In this regard, the sensitivity of free nucleons and LCPs with MSTB towards the neutron-rich system can also play an important role in the prediction of symmetry energy, which is a topic of separate discussion. After assessing the sensitivity, for the further study, the isospin sensitive free nucleons and LCPs, namely, n, p, ^{3}H and ^{3}He are used.

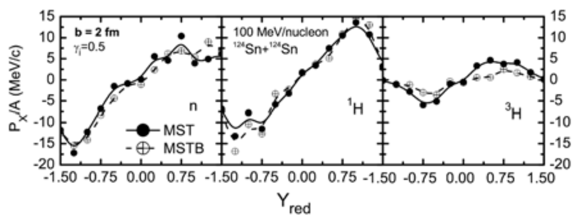

Fig.2 Importance of MSTB in flow calculations by plotting the rapidity distribution dependence of P_x/A for neutrons, protons, ^{3}H particles with MST and MSTB algorithms at 100 MeV/nucleon for ^{124}Sn+^{124}Sn reaction system.

Using the first method, we have plotted in Fig.2, the rapidity distribution of P_x/A for neutrons, protons and ^{3}H particles at 100 MeV/nucleon for the central collisions of ^{124}Sn reaction system. The mid-rapidity slope of P_x/A represents the directed flow. The flow is found to be affected only for the heavier fragments, neutrons and protons flow is least affected. This is due to the isotropic (anisotropic) distribution of free nucleons (LCPs)[37]. Naturally, yield has to be independent of the type of distribution, while momentum of particles is supposed to depend strongly on the type of distribution. One of the examples is that different behavior of flow is obtained in earlier studies for different distribution regions[42,43]. The sensitivity of flow with MSTB towards the isospin physics at higher incident energy is presented in the following sections.

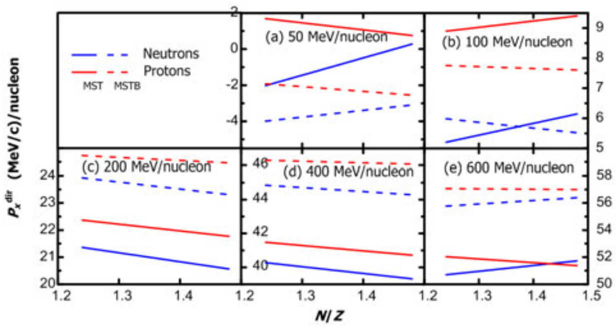

Fig.3 (Color online) Isospin asymmetry dependence of directed flow for isobaric pair neutrons, protons with MST and MSTB method. The different panels are at different incident energy ranging from 50 to 600 MeV/nucleon.

In order to study the role of MSTB at higher incident energies and towards isospin physics, in Fig.3, the isospin asymmetry dependence of more reliable flow quantity $P_x^{\rm dir}$ is plotted for isobaric pairs n, p (^{3}H,^{3}He) with MST and MSTB algorithm from 50 to 600 MeV/nucleon. In Fig.3, with increase in the incident energy, isospin asymmetry dependence of directed flow for n, p is found to affect more by the method of clusterization in comparison with the type of particles. In all panels, protons (neutrons) have more (less) positive directed flow with MST as well as MSTB method. This is due to the contribution of Coulomb repulsion in protons production. The directed flow of protons as well as neutrons becomes more positive with MSTB over MST towards the high incident energy. This is true because there was increased content of unstable fragments with MST towards the high incident energy. The MSTB will break these unstable fragments into free particles (as per the condition specified earlier) and hence dominance of collisions will take place. Moreover, higher positive flow of neutrons towards the high incident energy can act as a tool for isospin dependent cross section. Furthermore, these findings are suggesting a shift in the balance energy with different kind of fragments.

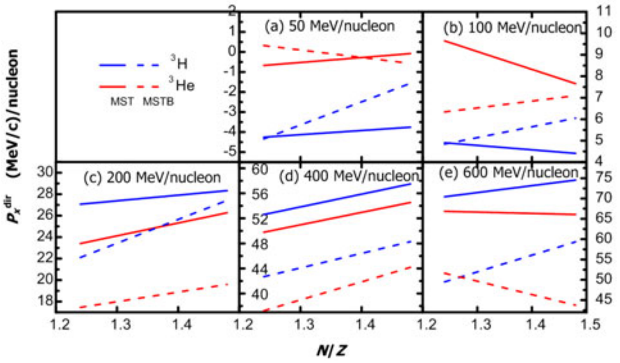

Fig.4 (Color online) Same as in Fig.3, but for the ^{3}H and ^{3}He particles.

In Fig.4, just like *n* and *p*, the isospin asymmetry dependence of directed flow for ^{3}H, ^{3}He is found to be affected more by the method of clusterization compared to type of particles towards the high incident energy. Up to 100 MeV/nucleon, ^{3}He (^{3}H) particles have more (less) positive directed flow, but, at and after 200 MeV/nucleon, ^{3}H (^{3}He) particles have more (less) positive flow with MST as well as MSTB methods. This is due to the dominance of mean field (collisions) at and below (at and above) 100 (200) MeV/nucleon. When the mean field (collisions) dominates, Coulomb interactions （isospin dependent cross sections）make the flow more positive for proton rich (neutron-rich) ^{3}He (^{3}H) particles. Towards the higher incident energies, the flow of ^{3}H as well as ^{3}He becomes more positive with MST over MSTB method. As with the inclusion of binding energy restrictions in clusterization algorithm, most of the ^{3}H and ^{3}He were found unstable. Especially, this is true for ^{3}He and hence directed flow is decreased with MSTB, which has resulted in the increase of the transverse directed flow for neutrons and protons (as discussed before in Fig.3).

In brief, the directed flow of isospin sensitive particles *n*, *p*, ^{3}H, ^{3}He is affected by the different method of clusterization. The role of different clusterzation method is dominating for the isobaric pair (^{3}H, ^{3}He) compared to (*n*, *p*). Moreover, the behavior of directed flow for (*n*, *p*) pair is smooth with incident energy, but， the transition is observed for the directed flow of (^{3}H, ^{3}He) pair with incident energy. The isobaric pair (*n*, *p*) and (^{3}H, ^{3}He) has opposite sensitivity with the type of method of clusterization. These points suggest that the methods of clusterization are going to play an important role in understanding the isospin physics and hence need to be handled carefully during the determination of symmetry energy also.

4 Conclusion

The modified clusterization method has a time saving advantage in terms of providing the real, bound and stable fragments at quite early time. It has the capability to obtain the results similar to the complicated secondary algorithms SACA and ECRA. Moreover, the complicated methods have the drawback over MSTB of time consuming and consists a lot of adjustable parameters. The directed flow of isobaric pair ^{3}H-^{3}He in comparison to *n*-*p* is found to be strongly dependent on the isospin asymmetry caused by MSTB with the increase of incident energy as well as isospin of the reaction systems. This indicates it as a robust approach for understanding the high density behavior of isospin physics or symmetry energy.

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