ac Josephson effect in the resonant tunneling through mesoscopic superconducting junctions

Yu Zhu[1], Qing-feng Sun[2], Wei Li[1], Zhong-shui Ma[1], and Tsung-han Lin[1*]

[1]*State Key Laboratory for Mesoscopic Physics and*

Department of Physics, Peking University, Beijing

100871, China

[2]*Center for the Physics of Materials and*

Department of Physics, McGill University, Montreal,

PQ, Canada H3A 2T8

Abstract

We investigate ac Josephson effect in the resonant tunneling through mesoscopic superconducting junctions. In the presence of microwave irradiation, we show that the trajectory of multiple Andreev reflections can be closed by emitting or absorbing photons. Consequently, photon-assisted Andreev states are formed and play the role of carrying supercurrent. On the Shapiro steps, dc component appears when the resonant level E_0 is near the position $V_L + \frac{1}{2}m_1\omega = V_R + \frac{1}{2}m_2\omega$ ($e = \hbar = 1$), where m_1 and m_2 are integers, ω is the frequency of microwave, V_L and V_R the chemical potentials of the superconducting leads. Analytical result is derived in the limit $\Delta \to \infty$, based on which new features of ac Josephson effect are revealed.

Introduction–Since the achievement of atom-size superconducting quantum point contact (SQPC) by break junction technique [1–4], considerable theoretical works have been devoted to the transport problem in S-I-S or S-N-S structures (S=superconductor, I=tunnel barrier, N=normal metal). Comparison between experimental data and theoretical calculation suggests that the coherent picture of multiple Andreev reflection (MAR) is the central conception in understanding the transport through such mesoscopic SQPC. Nevertheless, less attention is paid to the resonant tunneling (especially in ac transport) through mesoscopic superconducting junctions, in which the central region is consist of discrete electronic states. Such resonant tunneling can be achieved in nanoparticles sandwiched between superconducting membranes [5], or metallic clusters absorbed on the STM substrate [6], or a piece of carbon nanotube tunnel coupled to superconducting electrodes [7], or quantum dots fabricated in the S/2DEG hybrid structures [8], etc.

The transport through these structures is greatly modified due to the existence of discrete energy levels in the central region. It is shown by Yeyati *et al.* [9] and Johansson *et al.* [10] that the combination of MAR processes and resonant transmission gives rise to a rich subharmonic gap structure (SGS). Let us consider the transport through a resonant level with width Γ coupled to S leads with superconducting gap Δ. In the limit $\Gamma \gg \Delta$, the resonance is sufficient broadened such that the I-V characteristics mimic that of SQPC. In the regime $\Gamma \sim \Delta$, pronounced SGS appears with concomitant negative differential conductance, which is distinguished from the nonresonant transport through SQPC. In the limit $\Gamma \ll \Delta$, although SGS is more particular in the log-scale plot, the tunnel current as a whole is exponentially small. This is because a narrow resonance can not cover all the MAR trajectory, and it is very unlikely that multiple discrete levels are just in the position where MAR trajectory passes (see Fig.1a).

In this paper, we address the effect of microwave (MW) irradiation on the resonant tunneling through mesoscopic superconducting junctions. We show that the tunnel current is greatly enhanced on the Shapiro steps, when the resonant level E_0 is near the position $V_L + \frac{1}{2}m_1\omega = V_R + \frac{1}{2}m_2\omega$ ($e = \hbar = 1$ throughout), where m_1 and m_2 are integers, ω is the frequency of MW, V_L and V_R are the chemical potentials of left and right S lead. This can be attributed to the formation of photon-assisted Andreev states (PAAS), which play the role of carrying supercurrent. In the limit $\Delta \to \infty$, analytical results are derived, revealing new features of ac Josephson effect in the case of resonant tunneling.

Hamiltonian and Formulation–The model Hamiltonian reads $H = \sum_{\beta=L,R} H_\beta + H_{cent} +$

H_T, in which $H_\beta = \sum_{k\sigma} \varepsilon_k a^\dagger_{\beta k \sigma} a_{\beta k \sigma} + \sum_k (\Delta e^{i\phi_\beta} a^\dagger_{\beta k \uparrow} a^\dagger_{\beta - k \downarrow} + H.c.)$ is the Hamiltonian for the βth S lead, $H_{cent} = E_0 \sum_\sigma c^\dagger_\sigma c_\sigma$ the resonant level in the central region, and $H_T = \sum_{\beta k \sigma} (v_\beta \exp \left[i \int_0^t \tilde{V}_\beta(t') dt' \right] a^\dagger_{\beta k \sigma} c_\sigma + H.c.)$ the tunnel couplings. $\tilde{V}_\beta(t) \equiv V_\beta + W_\beta \cos \omega t$ is the time-dependent voltage drop across the βth barrier, where V_β is the chemical potential controlled by dc bias voltage and $W_\beta \cos \omega t$ the ac voltage induced by MW irradiation. For simplicity, we assume that Coulomb interaction and multi-level effect can be ignored in the central region. This assumption is somehow too ideal to achieve, but allows us to obtain analytical results and therefore instructive for the understanding of more complicated cases.

Keldysh Green functions are defined in the 2×2 Nambu representation:

$$G^{r,a,<}_{t_1 t_2} \equiv \begin{pmatrix} \langle\langle c_\uparrow(t_1) | c^\dagger_\uparrow(t_2) \rangle\rangle^{r,a,<} & \langle\langle c_\uparrow(t_1) | c_\downarrow(t_2) \rangle\rangle^{r,a,<} \\ \langle\langle c^\dagger_\downarrow(t_1) | c^\dagger_\uparrow(t_2) \rangle\rangle^{r,a,<} & \langle\langle c^\dagger_\downarrow(t_1) | c_\downarrow(t_2) \rangle\rangle^{r,a,<} \end{pmatrix}, \tag{1}$$

The time-dependent current flowing out of the βth lead can be expressed as

$$I_\beta(t) = 2 \operatorname{Re} Tr \sigma_z (G^r \circ \Sigma^<_\beta + G^< \circ \Sigma^a_\beta)_{tt}, \tag{2}$$

in which σ_z is the 3rd Pauli matrix, $\circ$ the shorthand notation for integration over intermediate time variable, and Σ_β the self energy due to tunnel coupling between the central region and the βth lead. G^r, G^a, and $G^<$ satisfy the integral equation:

$$G^{r,a} = g^{r,a} + g^{r,a} \circ \Sigma^{r,a} \circ G^{r,a}, \tag{3}$$

$$G^< = G^r \circ \Sigma^< \circ G^a, \tag{4}$$

in which

$$g^{r,a}_{t_1 t_2} = \int \frac{d\epsilon}{2\pi} e^{-i\epsilon(t_1 - t_2)} g^{r,a}(\epsilon), \tag{5}$$

$$\Sigma^{r,a,<}_{\beta, t_1 t_2} = U_\beta(t_1) \int \frac{d\epsilon}{2\pi} e^{-i\epsilon(t_1 - t_2)} \Sigma^{r,a,<}_\beta(\epsilon) U^\dagger_\beta(t_2), \tag{6}$$

where

$$g^{r,a}(\epsilon) = \begin{pmatrix} \frac{1}{\epsilon - E_0 \pm i0^+} & 0 \\ 0 & \frac{1}{\epsilon + E_0 \pm i0^+} \end{pmatrix}, \tag{7}$$

$$\Sigma^{r,a}_\beta(\epsilon) = \mp \frac{i}{2} \Gamma_\beta \frac{\epsilon \pm i\eta}{\sqrt{(\epsilon \pm i\eta)^2 - \Delta^2}} \begin{pmatrix} 1 & -\frac{\Delta}{\epsilon \pm i\eta} e^{-i\phi_\beta} \\ -\frac{\Delta}{\epsilon \pm i\eta} e^{+i\phi_\beta} & 1 \end{pmatrix}, \tag{8}$$

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$$\Sigma_\beta^<(\epsilon) = f(\epsilon)\left[\Sigma_\beta^a(\epsilon) - \Sigma_\beta^r(\epsilon)\right], \tag{9}$$

$$U_\beta(t) = \begin{pmatrix} \exp\left[-i\int_0^t \tilde{V}_\beta(t')dt'\right] & 0 \\ 0 & \exp\left[+i\int_0^t \tilde{V}_\beta(t')dt'\right] \end{pmatrix}, \tag{10}$$

with Γ_β being the coupling strength, η the dephasing rate in the S lead, $f(\epsilon)$ the Fermi distribution function, and $\text{Im}\sqrt{z} > 0$ as a convention. The remaining task is to solve these integral equations and evaluate the dc component of the time-dependent current.

There are two intrinsic frequencies in the problem, $\omega_1 = 2V = 2(V_L - V_R)$ and $\omega_2 = \omega$. Generally, one may perform a Fourier transform

$$A_{t_1 t_2} = \sum_{l_1 l_2} e^{i(l_1\omega_1 + l_2\omega_2)t_1} \int \frac{d\epsilon}{2\pi} e^{-i\epsilon(t_1-t_2)} \tilde{A}_{l_1 l_2}(\epsilon), \tag{11}$$

and derive the recursive relation for $\tilde{A}_{l_1 l_2}(\epsilon)$, as done in [11]. The calculation in this way relies on the numerical computing power, and analytical results are impossible. We note, however, that in the case of narrow resonance, dc component appears only if $\omega_1 = N\omega_2$ with N being an integer. This becomes clear by considering PAAS shown in Fig.1b and 1c: an electron (a hole) incomes through the resonant level E_0 and Andreev reflected by the right superconducting gap, the reflected hole (the reflected electron) exchanges m_1 photons with the MW field so that it can again passes through E_0. Then the hole (the electron) is Andreev reflected by the right superconducting gap as an electron (a hole), and exchanges m_2 photons to close the trajectory (detailed discussion for photon-assisted Andreev reflection is available in [12]). It is easy to see that the formation of PAAS requires $2V = (m_1 + m_2)\omega$. For this reason, we shall only consider the case of $V = V_N = \frac{N}{2}\omega$, while the current deviated from this condition is negligible small.

The problem is largely simplified since ω can be used as the basic frequency in the Fourier expansion. Define the Fourier transformation as

$$A_{t_1 t_2} = \sum_l e^{il\omega t_1} \int \frac{d\epsilon}{2\pi} e^{-i\epsilon(t_1-t_2)} \tilde{A}_l(\epsilon), \tag{12}$$

$$\mathbf{A}_{mn}(\epsilon) = \tilde{A}_{m-n}(\epsilon - n\omega). \tag{13}$$

The definition guarantees the nice property that if $C = A \circ B$ then $\mathbf{C}_{mn}(\epsilon) = \sum_k \mathbf{A}_{mk}(\epsilon)\mathbf{B}_{kn}(\epsilon)$. The Fourier transformed $g^{r,a}$ and $\Sigma_\beta^{r,a,<}$ are

$$\mathbf{g}_{mn}^{r,a}(\epsilon) = \delta_{mn} g^{r,a}(\epsilon - n\omega), \tag{14}$$

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$$\mathbf{\Sigma}^{r,a,<}_{R,mn}(\epsilon) = \sum_l \begin{pmatrix} J_{l-m}(\alpha_R)\Sigma^{r,a,<}_{R,11}(\epsilon^0_l)J_{l-n}(\alpha_R) & J_{l-m}(\alpha_R)\Sigma^{r,a,<}_{R,12}(\epsilon^0_l)J_{n-l}(\alpha_R) \\ J_{m-l}(\alpha_R)\Sigma^{r,a,<}_{R,21}(\epsilon^0_l)J_{l-n}(\alpha_R) & J_{m-l}(\alpha_R)\Sigma^{r,a,<}_{R,22}(\epsilon^0_l)J_{n-l}(\alpha_R) \end{pmatrix}, \tag{15}$$

$$\mathbf{\Sigma}^{r,a,<}_{L,mn}(\epsilon) = \sum_l \begin{pmatrix} J_{l-m}(\alpha_L)\Sigma^{r,a,<}_{L,11}(\epsilon^-_l)J_{l-n}(\alpha_L) & J_{l-m-N}(\alpha_L)\Sigma^{r,a,<}_{L,12}(\epsilon^+_l)J_{n-l}(\alpha_L) \\ J_{m-l-N}(\alpha_L)\Sigma^{r,a,<}_{L,21}(\epsilon^-_l)J_{l-n}(\alpha_L) & J_{m-l}(\alpha_L)\Sigma^{r,a,<}_{L,22}(\epsilon^+_l)J_{n-l}(\alpha_L) \end{pmatrix}, \tag{16}$$

in which $\alpha_\beta \equiv \frac{W_\beta}{\omega}$ is the MW strength on the βth tunnel barrier, $J_n(x)$ the nth Bessel function, $\epsilon^0_l \equiv \epsilon - l\omega$ and $\epsilon^\pm_l \equiv \epsilon - l\omega \pm V_N$, $V_L = V_N$ and $V_R = 0$ are set as a convention. Correspondingly, the equations for G^r, G^a, and $G^<$ are Fourier transformed into $\mathbf{G}^{r,a}(\epsilon) = \mathbf{g}^{r,a}(\epsilon) + \mathbf{g}^{r,a}(\epsilon)\mathbf{\Sigma}^{r,a}(\epsilon)\mathbf{G}^{r,a}(\epsilon)$ and $\mathbf{G}^<(\epsilon) = \mathbf{G}^r(\epsilon)\mathbf{\Sigma}^<(\epsilon)\mathbf{G}^a(\epsilon)$, or equivalently,

$$\mathbf{G}^{r,a}(\epsilon) = \mathbf{g}^{r,a}(\epsilon) + \mathbf{g}^{r,a}(\epsilon)\mathbf{\Sigma}^{r,a}(\epsilon)\mathbf{g}^{r,a}(\epsilon) + \mathbf{g}^{r,a}(\epsilon)\mathbf{\Sigma}^{r,a}(\epsilon)\mathbf{g}^{r,a}(\epsilon)\mathbf{\Sigma}^{r,a}(\epsilon)\mathbf{g}^{r,a}(\epsilon) + \cdots, \tag{17}$$

$$\mathbf{G}^<(\epsilon) = [\mathbf{g}^r(\epsilon) + \mathbf{g}^r(\epsilon)\mathbf{\Sigma}^r(\epsilon)\mathbf{g}^r(\epsilon) + \cdots]\,\mathbf{\Sigma}^<(\epsilon)\,[\mathbf{g}^a(\epsilon) + \mathbf{g}^a(\epsilon)\mathbf{\Sigma}^a(\epsilon)\mathbf{g}^a(\epsilon) + \cdots]. \tag{18}$$

We note that finite order perturbation expansion is inadequate in the problem, because the formation of PAAS involves up to infinite order of tunneling processes. To re-sum up the series, we adopt the resonant approximation [12]

$$\frac{1}{(\epsilon - l_1\omega \pm E_0 + i0^+)} \cdot \frac{1}{(\epsilon - l_2\omega \pm E_0 + i0^+)} \approx \delta_{l_1 l_2}\frac{1}{(\epsilon - l_1\omega \pm E_0 + i0^+)^2} \cdot \tag{19}$$

The approximation implies that the overlap between sidebands $E_0 + l_1\omega$ and $E_0 + l_2\omega$ can be ignored if $l_1 \neq l_2$, which is justified when $\Gamma_\beta \ll \omega$. Applying the approximation to Eq.(17) and Eq.(18), one can obtain the solution

$$\mathbf{G}^{r,a,<}_{mn}(\epsilon) = \begin{pmatrix} \delta_{mn}\sum_k \widehat{G}^{r,a,<}_{mk,11} & \widehat{G}^{r,a,<}_{mn,12} \\ \widehat{G}^{r,a,<}_{nm,21} & \delta_{mn}\sum_k \widehat{G}^{r,a,<}_{kn,22} \end{pmatrix}, \tag{20}$$

in which

$$\widehat{G}^{r,a}_{mn} = \left[(\widehat{g}^{r,a}_{mn})^{-1} - \widehat{\Sigma}^{r,a}_{mn}\right]^{-1}, \tag{21}$$

$$\widehat{G}^<_{mn} = \widehat{G}^r_{mn}\widehat{\Sigma}^<_{mn}\widehat{G}^a_{mn}, \tag{22}$$

with

$$\widehat{g}^{r,a}_{mn} = \begin{pmatrix} \mathbf{g}^{r,a}_{mm,11}(\epsilon) & 0 \\ 0 & \mathbf{g}^{r,a}_{nn,22}(\epsilon) \end{pmatrix}, \tag{23}$$

$$\widehat{\Sigma}^{r,a,<}_{mn} = \begin{pmatrix} \mathbf{\Sigma}^{r,a,<}_{mm,11}(\epsilon) & \mathbf{\Sigma}^{r,a,<}_{mn,12}(\epsilon) \\ \mathbf{\Sigma}^{r,a,<}_{nm,21}(\epsilon) & \mathbf{\Sigma}^{r,a,<}_{nn,22}(\epsilon) \end{pmatrix}. \tag{24}$$

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It can be shown that the relations $\mathbf{G}^r = (\mathbf{G}^a)^\dagger$ and $\mathbf{G}^< = -(\mathbf{G}^<)^\dagger$ still hold in the solution.

With these Green functions, the time-dependent current can be expressed in a summation over ac components

$$I_\beta(t) = 2\,\mathrm{Re}\sum_l e^{\mathrm{i}l\omega t}\left\{\int \frac{d\epsilon}{2\pi}Tr\sigma_z\left[\mathbf{G}^r(\epsilon)\mathbf{\Sigma}_\beta^<(\epsilon) + \mathbf{G}^<(\epsilon)\mathbf{\Sigma}_\beta^a(\epsilon)\right]_{l0}\right\}. \qquad (25)$$

The current formula can be applied to the study of ac harmonics. However, we are more interested in the dc component $\bar{I} = \bar{I}_L = -\bar{I}_R$, due to the experimental reasons. To produce analytical results, we take the limit $\Delta \to \infty$, due to which all single particle processes are forbidden and Andreev reflection is the only conducting mechanism. After some algebra and take the limit $\eta \to 0$ (notice that $\lim_{\eta\to 0}\frac{\eta}{x^2+\eta^2} = \pi\delta(x)$), one can derive

$$\bar{I} = 2\sin\phi\frac{\Gamma_L\Gamma_R}{4}(-)\sum_m J_{m-N}(2\alpha_L)J_m(2\alpha_R)\frac{1}{(E_m^+ - E_m^-)^2}\left[F(E_m^+) + F(E_m^-)\right], \qquad (26)$$

$$F(\epsilon) = \left(\epsilon + E_0 - \frac{1}{2}m\omega\right)\sum_l\left[f\left(\epsilon_l^0 + \frac{1}{2}m\omega\right)J_l^2(\alpha_R) + f\left(\epsilon_l^- + \frac{1}{2}m\omega\right)J_l^2(\alpha_L)\right] + \qquad (27)$$
$$\left(\epsilon - E_0 + \frac{1}{2}m\omega\right)\sum_l\left[f\left(\epsilon_l^0 - \frac{1}{2}m\omega\right)J_{-l}^2(\alpha_R) + f\left(\epsilon_l^+ - \frac{1}{2}m\omega\right)J_{-l}^2(\alpha_L)\right].$$

in which $E_m^\pm = \pm\sqrt{\left(E_0 - \frac{1}{2}m\omega\right)^2 + |\Gamma_m|^2}$ and $\Gamma_m = \frac{1}{2}\left[\Gamma_R J_m(2\alpha_R) + \Gamma_L e^{-\mathrm{i}\phi}J_{m-N}(2\alpha_L)\right]$. Eq.(26) is the central result of this paper, which is for the dc component of Josephson current in the resonant tunneling through mesoscopic superconducting junctions.

Before numerical study, we make a few remarks on this result: (1) The phase dependence of $\bar{I}$ is mainly determined by the prefactor $\sin\phi$, and a weak $\cos\phi$ dependence is hidden in $E_m^\pm$ via Γ_m. For this reason, ϕ is set as $\frac{\pi}{2}$ in the numerical calculation. (2) For the special case $N = 0$, $\alpha_L = \alpha_R = 0$, and $\Gamma_L = \Gamma_R = \Gamma$, $\bar{I} = 2\sin\phi\frac{\Gamma^2}{4}\frac{1}{\tilde{E}_0}\left[f\left(-\tilde{E}_0\right) - f\left(+\tilde{E}_0\right)\right]$, with $\tilde{E}_0 = \sqrt{E_0^2 + \left(\Gamma\cos\frac{\phi}{2}\right)^2}$, which reproduces the exact result for dc Josephson current through a resonant level. (3) The current formula is for the gauge choice of $V_L = V_N$ and $V_R = 0$. It is easy to see that the formula is invariant under the transformation $L \longleftrightarrow R$, $V_N \to -V_N$, $N \to -N$, and $E_0 \to E_0 - V_N$, which corresponds to the gauge choice $V_L = 0$ and $V_R = -V_N$. (4) $E_m^\pm$ are the poles of $\mathbf{G}^r(\epsilon)$, which can be interpreted as PAAS. Obviously, $E_m^\pm$ contribute to the supercurrent with opposite signs. One can expect that resonant structures will appear near $E_0 = \frac{1}{2}m\omega$. (5) Bessel functions enter the current formula not in the square form $J_n^2(x)$. It can be shown that $\bar{I}(\alpha_L, \alpha_R, \phi) = \bar{I}(-\alpha_L, -\alpha_R, \phi)$ for N even and $\bar{I}(\alpha_L, \alpha_R, \phi) = \bar{I}(-\alpha_L, -\alpha_R, \phi + \pi)$ for N odd.

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numerical results–Firstly, we discuss the case that MW is applied symmetrically to the left and right tunnel barrier, i.e., $\alpha_L = \alpha_R$. Fig.2 shows the curves of $I_c = \bar{I}(\phi = \frac{\pi}{2})$ vs $E_0 - \frac{V_N}{2}$ at bias voltage $V_N = \frac{N}{2}\omega$, with $N = 0, 1, 2, 3, 4$ (the curves are shifted by $\frac{V_N}{2}$ for demonstration). Four features are noteworthy in the plot: (1) The curve is symmetric (anti-symmetric) with respect to $E_0 = \frac{V_L + V_R}{2} = \frac{V_N}{2}$ for N even (odd). (2) Photon-assisted structures appear near $E_0 = V_L - \frac{1}{2}(N - m)\omega = V_R + \frac{1}{2}m\omega$. (3) There are two types of resonant structures, single peak (dip) and peak-dip pair. (4) The structures grow with the MW strength non-monotonously.. Feature (1) is due to the relation $I(E_0, \phi) = I(V_N - E_0, \phi)$ for N even and $I(E_0, \phi) = I(V_N - E_0, \phi + \pi)$ for N odd. Using $I(E_0, -\phi) = -I(E_0, \phi)$, one can obtain $I(E_0, \phi = \frac{\pi}{2}) = (-)^N I(V_N - E_0, \phi = \frac{\pi}{2})$. Feature (2) can be understood in terms of PAAS shown in Fig.1b and 1c: Electron and hole are Andreev reflected back and forth by the superconducting gaps. When E_0 is near the position $V_L - \frac{1}{2}(N - m)\omega = V_R + \frac{1}{2}m\omega$, quasiparticles may exchange $N - m$ photons with the MW field at the left tunnel barrier and exchange m photons at the right, so that the trajectory is closed and bound states are formed. It is these PAAS that play the role of carrying supercurrent. Understanding of feature (3) and (4) needs quantitative analysis of each photon-assisted structure. To proceed, we decompose the total current into $\bar{I} = \sum_m I_m$, and expand I_m near $E_0 = \frac{1}{2}m\omega$. Let $E_0 = \frac{1}{2}m\omega + \delta$, one can obtain $I_m = I_m^+ + I_m^-$ with

$$I_m^{\pm} = 2\sin\phi \frac{\Gamma^2}{4}(-)J_{m-N}(2\alpha_L)J_m(2\alpha_R)\frac{1}{4\left(\delta^2 + |\Gamma_m|^2\right)}$$
$$\left[\alpha_m^{\pm}\left(\pm\sqrt{\delta^2 + |\Gamma_m|^2} + \delta\right) + \beta_m^{\pm}\left(\pm\sqrt{\delta^2 + |\Gamma_m|^2} - \delta\right)\right]. \tag{28}$$

The cancelation between I_m^+ and I_m^- results in two types of resonant structures: $C_1 \frac{1}{\sqrt{\delta^2 + |\Gamma_m|^2}} + C_2 \frac{\delta}{\delta^2 + |\Gamma_m|^2}$, corresponding to single peak (dip) and peak-dip pair. At zero temperature, C_1 and C_2 are proportional to

$$C_1 \propto \sum_l \left[\delta_{l, \frac{1}{2}m}J_l^2(\alpha_R) + \delta_{l, \frac{1}{2}(m-N)}J_l^2(\alpha_L)\right], \tag{29}$$

$$C_2 \propto \left[\sum_{l > -\frac{1}{2}m} J_l^2(\alpha_R) - \sum_{l > \frac{1}{2}m} J_l^2(\alpha_R) + \sum_{l > -\frac{1}{2}(m-N)} J_l^2(\alpha_L) - \sum_{l > \frac{1}{2}(m-N)} J_l^2(\alpha_L)\right], \tag{30}$$

The oscillatory nature of Bessel functions in these coefficients is responsible for feature (4). There is an interesting special case, $N = 2$, $m = 1$, $\alpha_L = \alpha_R$, due to which $C_1 = C_2 = 0$ for arbitrary MW strengths. Correspondingly, the resonant structure near $E_0 - \frac{V_N}{2} = 0$ is always missing for $N = 2$.

Next, we investigate the case that MW is applied only to one of the tunnel barriers, i.e., $\alpha_L = 0$ and $\alpha_R \neq 0$. Fig.3 shows the curves of I_c vs E_0 for $\alpha_L = 0$ and $\alpha_R = 1$. In contrast to the symmetric case, a single peak is pinned at $E_0 = V_L$. The reason is as follows: since no MW is applied to the left barrier, Andreev tunneling through this barrier occurs only when E_0 lines up with V_L, while photon-assisted processes is allowed at the right barrier irradiated by MW field. The inset shows I_c vs MW strength α_R at $E_0 = V_L$ for $N > 0$ (The case of $N < 0$ can be easily deduced from $N > 0$). One can see in the plot that the peak height (including the sign) is predominated by the prefactor $J_{m-N}(2\alpha_L)J_m(2\alpha_R)$. For $\alpha_L = 0$, $J_{m-N}(2\alpha_L)$ requires $m = N$, and the peak height is proportionate to $J_N(2\alpha_R)$. We note that there exist some regions of MW strength where I_c and V_N have opposite signs, which can be viewed as quantum pump effect. However, this feature is dramatically different from the quantum pump effect in normal mesoscopic junctions.

conclusion–To sum up, we have investigated ac Josephson effect in the resonant tunneling through mesoscopic superconducting junctions. We show that PAAS play an essential role for conducting supercurrent through a narrow resonance, in which MAR trajectory is closed by exchanging photons with MW field. On the Shapiro steps $2V = N\omega$, dc component appears when the resonant level E_0 is near $V_L - (N - m)\omega = V_R + m\omega$, due to the formation of PAAS. In the limit $\Delta \to \infty$, analytical result, Eq.(26), is derived for the dc component of Josephson current, and help to understand new features of ac Josephson effect in the case of resonant tunneling. We have dropped Coulomb interaction in the calculation, the derived results are meaningful in the following senses: (1) They are directly applicable to the systems where Coulomb blockade effect is negligible, i.e., $U \sim \Gamma \ll \Delta$ (U is the charging energy). This is possible by using substrate with large dielectric constant to reduce U or using proper material as S lead to obtain large Δ. (2) They are instructive for more complicated cases. We note that the conception of PAAS is also useful for the interacting case. For $U \gg \Gamma$, the resonant level E_0 is effectively split into two resonances E_0 and $E_0 + U$. Similar to Fig.1b and 1c, one can draw diagrams of closed MAR trajectory through these resonances. Moreover, Coulomb blockade can be partially removed by bias voltage or MW irradiation, as long as U is comparable to Δ. Obviously, calculation including interaction term is much more difficult, and analytical results are almost impossible. Efforts along this line are still in progress.

Mesoscopic Physics in Peking University.

* To whom correspondence should be addressed.

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FIGURE CAPTIONS

Fig. 1 Schematic diagram of the resonant tunneling through mesoscopic superconducting junctions. (a): Without MW irradiation, MAR trajectory can not be covered by a narrow resonance, and the tunnel current is exponentially small. (b) and (c): In the presence of MW irradiation, MAR trajectory can be closed by emitting or absorbing photons. Two types of PAAS are formed, carrying supercurrent with opposite signs.

Fig. 2 The curves of $I_c \equiv \bar{I}(\phi = \frac{\pi}{2})$ vs $E_0 - \frac{V_N}{2}$ at bias voltage $V_N = \frac{N}{2}\omega$ (N from 0 to 4), with symmetric MW strengths $\alpha_L = \alpha_R$. Parameters are: $\omega = 1$, $\Gamma_L = \Gamma_R = 0.02$, $k_B T = 0.001$. $V_L = V_N$ and $V_R = 0$ are set as a convention.

Fig. 3 The curves of $I_c \equiv \bar{I}(\phi = \frac{\pi}{2})$ vs E_0 at bias voltage $V_N = \frac{N}{2}\omega$ (N from -4 to 4), with MW strengths $\alpha_L = 0$ and $\alpha_R = 1$. Other parameters are the same as Fig.2. The inset shows I_c at $E_0 = V_L$ vs the MW strength α_R. Diagrams for corresponding PAAS is also shown in the plot.