On asymmetric brane creation

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We exhaust the brane instanton solutions—an Einstein brane inhabiting at different positions in a 5-dimensional Einstein bulk with negative curvature. We construct a brane instanton model consisting of a brane with asymmetric bulk along two sides of the brane. And the junction condition of the resulting space-time is analyzed in the frame of induced gravity (DGP model). In spirits of quantum gravity of path integral formulism we calculate the Euclidean actions on three canonical paths and then compare the Euclidean actions of different instantons per unit 4-volume. We also compare the Euclidean actions per unit 4-volume of instantons, which consist of a brane gluing to a fixed half, with other Euclidean actions of halves possessing different cosmological constants.

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I. INTRODUCTION

Quantum cosmology deals with the creation of our universe where and when the classical gravity theory may fail. In the formula

$$\Psi[\Sigma, h] = \sum_{M} \int D[g_E] \exp(-S_E[M, g_E]), \tag{1}$$

Ψ is the propagator from a 3-manifold (usually *nothing* taken) to (Σ, h), and S_E is the Euclidean action of the "path" M. The standard model of the quantum origin of the universe begins with a consisting of a compact, path connected, oriented, Riemannian manifold M_R adhered to a Lorentzian manifold M_L by a totally geodesic spacelike hypersurface Σ, which serves as an initial Cauchy surface for the Lorentzian evolution on manifold M_L. Given this setup we pass to the double $2M_R = M_R^z \cup M_R^y$ by joining two copies of M_R across Σ. This is a closed, path connected, oriented, Riemannian 4-manifold $M = 2M_R$ with a mirror isometry that fixes the totally geodesic submanifold Σ. Here M is a very generic topological manifold. The smaller set is often considered : the gravitational instantons, that is, M is an Einstein manifold [1]. Here we generalize the conception of instanton—we permit non mirror symmetry between M_R^z and M_R^y.

On the other hand the concept brane emerging recent years is very important in high energy physics and cosmology. In the brane world scenario, the standard model particles are confined to the 3-brane, while gravity can propagate in the whole space [2]. An inflating brane world with positive curvature and mirror symmetry created from "nothing" together with its Anti de Sitter (AdS) bulk has been considered in [3]. Then the creation of the inflationary brane universe in 5d bulk Einstein and Einstein-Gauss-Bonnet gravity has been analyzed in [4]. Brane instantons in F-theory have been explored in [5]. The relation between brane instanton and Taub-NUT in M-theory is studied in [6]. Brane instanton intersecting at sine angle is presented in [7]. Many different types of brane world creation models have been investigated widely [8]. The brane world models without mirror symmetry have been investigated to some extent: different black hole masses on the two sides on the brane [9], zero black hole mass and different cosmological constants on the two sides [10] and allowing for both types of generalizations [11]. The quantum creation of closed branes by totally

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antisymmetric tensor and gravity was treated as an interesting solution to cosmological problem in [12]. In this paper, we shall investigate the creation of a brane world model with non mirror symmetric bulk.

We study all the three types of solutions—negative curvature, positive curvature and Ricci flat Einstein branes inhabiting at different positions in a 5d negative curvature Einstein manifold and investigate instantons including brane without mirror symmetric in bulk in section II. Negative curvature Einstein manifold means $R = kg$, where R is Ricci tensor, g denotes metric and k is a negative real number; while positive curvature is characterized by a positive k. In section III we analyze the junction condition of the brane instanton solutions in the induced gravity frame. The main purpose of section IV is to obtain the actions of the different three types of branes, with or without induced gravity term so as to finding some clues of comparing their creation probabilities. Different from the symmetric case [3] we find the Gibbons-Hawking boundary term in context of asymmetric instanton must be considered. Finally we present our conclusions and discussions in section V.

II. INSTANTON SOLUTIONS

We consider a 4d brane embedded in a 5d Einstein bulk. In Gaussian normal coordinates the metric ansatz of a 5d Euclidean-Einstein space is written as

$$^{(5)}g_E = g_{ab}dx^a dx^b = dr^2 + b^2(r)ds_4^2. \tag{2}$$

Here Latin indices run from 0 to 4, $b(r)$ has the dimension of length. The induced 4-metric on the brane is $g_E = b^2(r)ds_4^2$. Then we introduce $h_E = ds_4^2$ for convenience.

There is an interesting relation between $^{(5)}g_E$ and g_E, as shown in the following lemma : *The bulk with metric ansatz (2) is a negative curvature Einstein manifold only if it has an Einstein submanifold.*

Pf: One selects an orthonormal base $e^r = dr, e^\mu = b\tilde{e^\mu}$, where $\tilde{e^\mu}$ is an orthonormal base of the h_E, Greek index labels 0–3. Under this base one has

$$^{(5)}R_{rr} = -\frac{4b''}{b}, \tag{3}$$

where prime stands for derivative with respect to r. If $^{(5)}g_E$ is an Einstein manifold, one has $^{(5)}R_{rr} = Cg_{rr}$, where C is a constant. Set $C = -\frac{4}{l^2}$ then one further has

$$l^2 b'' = b. \tag{4}$$

The general solution of the above equation is

$$b = c_1 e^{\frac{r}{l}} + c_2 e^{-\frac{r}{l}}, \tag{5}$$

where $c_1,\ c_2$ are two constants and l has the dimension of length. Along the directions of the submanifold we find

$$^{(5)}R_{\alpha\mu} = \frac{1}{b^2}[\tilde{R}_{\alpha\mu} - 3b'^2\delta_{\alpha\mu}] - \frac{1}{l^2}\delta_{\alpha\mu}. \tag{6}$$

If $^{(5)}g_E$ is an Einstein manifold, i.e., $^{(5)}R_{\alpha\mu} = -\frac{4}{l^2}\delta_{\alpha\mu}$, substituting (5) into (6), one has

$$\frac{1}{(c_1 e^{r/l} + c_2 e^{-r/l})^2}\left[\tilde{R}_{\alpha\mu} - \frac{3}{l^2}\left(c_1 e^{r/l} - c_2 e^{-r/l}\right)^2 \delta_{\alpha\mu}\right] - \frac{1}{l^2}\delta_{\alpha\mu} = -\frac{4}{l^2}\delta_{\alpha\mu}\ , \tag{7}$$

therefore one obtains

$$\tilde{R}_{\alpha\mu} = -\frac{3}{l^2}\delta_{\alpha\mu}(4c_1c_2)\ . \quad \square \tag{8}$$

We see this Ricci tensor $\tilde{R}_{\alpha\mu}$ can be treated as a solution of 4d vacuum Einstein equation with a cosmological constant $-4c_1c_2/l^2$. For giving prominence to the key point of this paper, we set

$$-4c_1c_2/l^2 = \epsilon, \tag{9}$$

where $\epsilon = -1,\ 0,\ 1$ denote negative curvature, Ricci flat or positive curvature canonical submanifold respectively. In this setup we have put the dimension of g_E into b^2, so both the components of h_E and the coordinates x^μ are dimensionless. Hence $\tilde{R}_{\alpha\mu}$ in (8) is dimensionless however $R_{\alpha\mu}(g_E)$ has dimension of [length]$^{-2}$. Simple calculation gives

$$R_{\alpha\mu}(g_E) = \epsilon b^{-2}\delta_{\alpha\mu}. \tag{10}$$

c_1 and c_2 have dimension of [length].

Here we point out the 3 types of hypersurfaces, i. e., the positive, negative and Ricci flat hypersurfaces correspond to different slicings of 5d Einstein manifold. To illuminate this point first we suppress 3d of the original 5d manifold, taking a cross section of the 5d manifold. Then we take a Wick rotation to return back to Lorentz manifold and then choose $c_1 = -c_2 = l/2,\ c_1 = c_2 = l/2$ and $c_1 = l,\ c_2 = 0$ to represent canonical positive curvature, negative curvature and Ricci flat submanifold respectively. If the space time we considered is not maximally symmetric it can not be imbedded in a 6d flat manifold. But we always can imbed it into a Ricci flat 6d manifold [13]. Either embedded into a 6d flat or 6d Ricci flat spacetime the cross section keeps the same. It is obvious the left 2d manifold is maximally symmetric manifold. We imbed this maximally symmetric manifold in the following 3d pseudo-Euclidean manifold

$$ds^2 = -dT^2 - dW^2 + dZ^2 \ , \tag{11}$$

where

$$-T^2 - W^2 + Z^2 = -l^2 \tag{12}$$

In the following parametrization

$$T = l\cosh(r_1/l)\sin(t_1) \ ,$$

$$W = l\cosh(r_1/l)\cos(t_1) \ ,$$

$$Z = l\sinh(r_1/l), \tag{13}$$

we have

$$g = dr_1^2 - l^2\cosh^2(r_1/l)dt_1^2 \ . \tag{14}$$

This chart covers the whole manifold except some singularities which form a zero measurement set. In this chart negative hypersurfaces stand at $r_1 = $ constant.

In the parametrization of

$$T = le^{r_2/l}t_2 \ ,$$

$$W = l\cosh(r_2/l) - \frac{1}{2}le^{r_2/l}t_2^2 \ ,$$

$$Z = l\sinh(r_2/l) + \frac{1}{2}le^{r_2/l}t_2^2 \ , \tag{15}$$

the metric becomes

$$g = dr_2^2 - l^2e^{2r_2/l}dt_2^2 \ . \tag{16}$$

This chart covers half of the manifold, namely, the region $Z + W > 0$. In this chart Ricci flat hypersurfaces stand at $r_2 = $ constant. The third parametrization is

$$T = l\sinh(r_3/l)\sinh(t_3) \ ,$$

$$W = l\cosh(r_3/l) \ ,$$

$$Z = l \sinh(r_3/l) \cosh(t_3), \tag{17}$$

by which we derive the metric

$$g = dr_3^2 - l^2 \sinh^2(r_3/l) dt_3^2 \ . \tag{18}$$

This chart covers half of the manifold, that is, the region $W > 0$. In this chart positive curvature hypersurfaces stand at $r_3 =$ constant. Now we present a simple conclusion which is not priorly obvious. We know there are 3 classes of parameterizations depending on the sign of c_1c_2. All the metrics of hypersurfaces in the same class can be written in standard form, given by (14), (16) or (18), simply by a coordinate transformation. Here we prove this conclusion in the positive curvature class. For any $c_1c_2 < 0$, define new coordinates t_5, r_5 by

$$\begin{aligned} T &= \frac{l}{\sqrt{-4c_1c_2}}(c_1e^{r_5/l} + c_2e^{-r_5/l}) \sinh t_5, \\ W &= \frac{l}{\sqrt{-4c_1c_2}}(c_1e^{r_5/l} - c_2e^{-r_5/l}), \\ Z &= \frac{l}{\sqrt{-4c_1c_2}}(c_1e^{r_5/l} + c_2e^{-r_5/l}) \cosh t_5. \end{aligned} \tag{19}$$

We find

$$g = dr_5^2 - b^2 dt_5^2 \ , \tag{20}$$

where

$$b = \frac{l}{\sqrt{-4c_1c_2}}(c_1e^{r_5/l} + c_2e^{-r_5/l}). \tag{21}$$

Considering the normalization condition (9) it is just (5). It is easy to find transformation between $(r_5,\ t_5)$ and $(r_3,\ t_3)$,

$$\begin{aligned} t_3 &= t_5 \\ \sinh(r_3/l) &= \frac{1}{\sqrt{-4c_1c_2}}(c_1e^{r_5/l} + c_2e^{-r_5/l}). \end{aligned} \tag{22}$$

Evidently for any $c_1,\ c_2$ in the same family, such as positive slicing, with a general conformal factor (21) the only work to get the standard chart is to rescale the coordinate r_5. Similar conclusions hold for the cases of negative and Ricci flat slicings. Therefore we always suppose that the metric of Einstein bulk together with its Einstein brane has been rescaled to the standard form (14), (16) or (18). This assumpotion carries many conveniences for the discussions of asymmetric instanton solutions.

If one only considers mirror symmetric instanton one can construct it by excising the spacetime region at $r > r_0$ left M_R and gluing two copies of the remaining spacetime along the 4-hypersurface at $r = r_0$, $M = M_R^z \bigcup M_R^y$. We consider the case where the bulk on two sides of the brane have no mirror symmetry, that is, the cosmological constants are different on the two sides of the brane. Because the 4-metric induced by the 5-metric of left bulk must be identical to the metric induced by right bulk, i.e., the 4-metric on the brane is unique, we have

$$g_E^{left} = g_E^{right}. \tag{23}$$

Obviously if g_E^{left} belongs to the positive curvature class but g_E^{right} belongs to the negative curvature class the bulk on two side cannot be glued together. So we consider to glue two halves of the bulk with sectional hypersurface in the same canonical class but different cosmological constants. For example to the brane in the positive curvature class the junction condition (23) becomes

$$l_1 \sinh(r_1/l_1) = l_2 \sinh(r_2/l_2), \tag{24}$$

where $l_1^2 = -6/{}^{(5)}\Lambda_L$ ($l_2^2 = -6/{}^{(5)}\Lambda_R$) is the characteristic length of the left (right) bulk and r_1 (r_2) the position of the brane in the left (right) bulk. The outline of an asymmetric instanton is shown in fig. 1.

Note that the maximally symmetric manifold—Minkowski, de Sitter or anti-de Sitter, is certainly Einstein manifold but we have many other choices, Schwarzschild (Schwarzschild-(Anti) de Sitter) solution, Kerr (Kerr-(Anti) de Sitter) solution etc. Here we note Randall-sundrum Minkowski brane is the flat solution included in Ricci flat class with $c_1c_2 = 0$, $c_1^2 + c_2^2 \neq 0$. Furthermore for convenience of later development we give (Anti) de Sitter manifold and Schwarzschild-((Anti) de Sitter) manifold as ds_4^2 clearly in Euclidean form. For (Anti) de Sitter

$$\text{de Sitter} \qquad ds_4^2 = d\chi^2 + \sin^2\chi d\Omega_{(3)}^2, \tag{25}$$

$$\text{anti de Sitter} \qquad ds_4^2 = d\chi^2 + \sinh^2\chi d\Omega_{(3)}^2, \tag{26}$$

where $d\Omega_{(3)}^2$ is 3-sphere. For Schwarzschild-((Anti-) de Sitter)

$$ds_4^2 = (1 - \frac{2m}{r'} + \epsilon r'^2)d\chi^2 + (1 - \frac{2m}{r'} + \epsilon r'^2)^{-1}dr'^2 + r'^2 d\Omega_{(2)}^2, \tag{27}$$

where r' is the radial coordinate on the brane and $d\Omega_{(2)}^2$ is 2-sphere. Kerr-(Kerr-(Anti) de Sitter) solution and any other Einstein manifold can be written similarly.

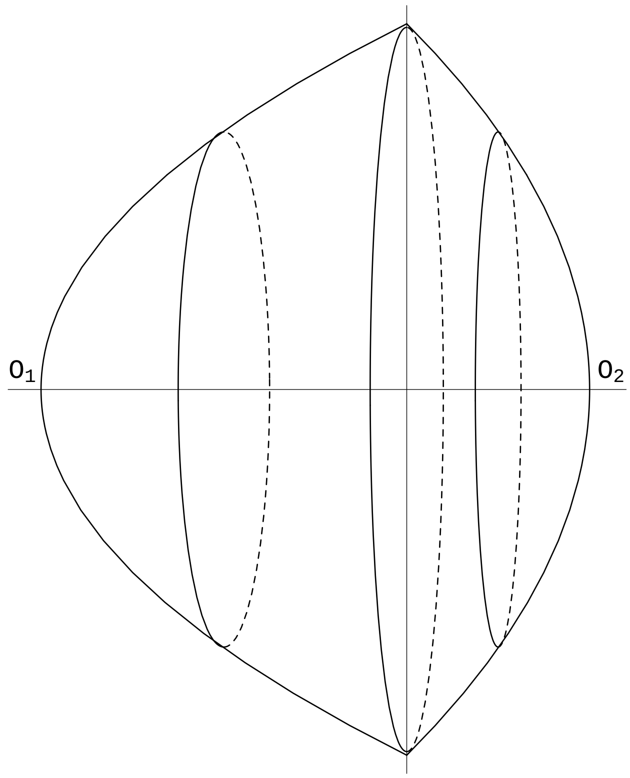

FIG. 1: Outline of an asymmetric instanton.

III. JUNCTION CONDITION

In this section we shall investigate the general junction in frame of brane–induced–gravity. The induced gravity brane model was proposed by Dvali et al. [14], and we work in the generalized Dvali-Gabadaze-Porrati model presented in [15].

We consider an asymmetric 5d gravitational instanton with a 4d brane on which an induced Ricci scalar term is confined,

$$S = S_{L\mathrm{bulk}} + S_{\mathrm{brane}} + S_{R\mathrm{bulk}}, \tag{28}$$

where

$$S_{L\mathrm{bulk}} = \int d^5X \sqrt{-\det({}^{(5)}g_L)} \left[\frac{1}{16\pi G_5} ({}^{(5)}R_L - 2^{(5)}\Lambda_L) + {}^{(5)}L_{L\mathrm{m}} \right], \tag{29}$$

$S_{R\mathrm{bulk}}$ can be written correspondingly, and

$$S_{\mathrm{brane}} = \int d^4x \sqrt{-\det(g)} \left[\frac{1}{8\pi G_5} K^{\pm} + L_{\mathrm{brane}}(g_{\alpha\beta}, \psi) \right]. \tag{30}$$

Here G_5 is the 5d gravitational constant, ${}^{(5)}R$ and ${}^{(5)}L_{\mathrm{m}}$ are the 5d scalar curvature and the matter Lagrangian in the bulk, respectively. A quantity with subscript L denotes it is valued in the left bulk and R, the right bulk. ${}^{(5)}\Lambda$ is the cosmological constant in the bulk. x^μ $(\mu = 0, 1, 2, 3)$ are the induced 4d coordinates on the brane, $K^\pm$ is the trace of extrinsic curvature on either side of the brane and $L_{\rm brane}(g_{\alpha\beta}, \psi)$ is the effective 4d Lagrangian, which is given by a generic functional of the brane metric $g_{\alpha\beta}$ and matter fields ψ.

Consider the brane Lagrangian

$$L_{\rm brane} = \frac{1}{16\pi G}(R - 2\lambda) + L_{\rm m}, \tag{31}$$

where λ is the cosmological constant on the brane, $L_{\rm m}$ denotes matter confined to the brane and R, G, the 4d scalar curvature and gravitational constant respectively. We assume that the 5d bulk space includes only a cosmological constant ${}^{(5)}\Lambda$. It is just a generalized version of the DGP model, which is obtained by setting $\lambda = 0$ as well as ${}^{(5)}\Lambda = 0$. The covariant equations in the case of ${}^{(5)}\Lambda_L = {}^{(5)}\Lambda_R$ have been obtained in [15], for ${}^{(5)}\Lambda_L \neq {}^{(5)}\Lambda_R$, the covariant equations were derived in [16].

A mirror symmetric closed brane-world instanton M can be constructed by excising the spacetime region at $r > r_0$ left M_R and gluing two copies of the remaining spacetime along the 4-hypersurface at $r = r_0$, $M = M_R^z \bigcup M_R^y$. We consider a more general class of instantons without mirror symmetry. Certainly the junction condition (23) must be satisfied. Furthermore the energy momentum tensor on the brane is constrained by the second fundamental form of the brane relative to the two sides of the bulk. The relation is Israel's junction condition

$$[K_{\mu\nu} - K g_{\mu\nu}]^\pm = 8\pi G_5 \tau_{\mu\nu}, \tag{32}$$

where $\tau_{\mu\nu}$ is the effective energy momentum stress tensor on the brane, $K_{\mu\nu}$ is the second fundamental form of the brane, $K = g_{\mu\nu}K^{\mu\nu}$, $[K_{\mu\nu} - K g_{\mu\nu}]^\pm = [K_{\mu\nu} - K g_{\mu\nu}]^+ - [K_{\mu\nu} - K g_{\mu\nu}]^-$ and $[K_{\mu\nu} - K g_{\mu\nu}]^+$ or $[K_{\mu\nu} - K g_{\mu\nu}]^-$ is the value of the expression at one side of the brane respectively. It is easy to get from (31),

$$\tau_{\mu\nu} = -\frac{1}{8\pi G}(\lambda g_{\mu\nu} + G_{\mu\nu}) - 2\frac{\delta L_m}{\delta g^{\mu\nu}} + g_{\mu\nu}L_m. \tag{33}$$

We omit matter term L_m in this section for the sake of instanton solution. From the lemma we have

$$R_{\mu\nu} = 3\epsilon b^{-2} g_{\mu\nu}. \tag{34}$$

One knows that b only depends on the fifth dimension in above equation. So in (33) the induced term $G_{\mu\nu}$ just acts as an cosmological constant from the brane view.

From (32) we arrive at

$$\widetilde{K}_{\mu\nu} = g_{\mu\nu} x \left(-\epsilon b^{-2} + \frac{\lambda}{3} \right), \tag{35}$$

where $x = \frac{G_5}{G}$.

Without announcement for a quantity Q, $\overline{Q} = \frac{1}{2}(Q^+ + Q^-)$, $\widetilde{Q} = Q^+ - Q^-$, Q^+ or Q^- is the value of the quantity at one side of the brane respectively. On the other hand

$$K_{\mu\nu} = \frac{1}{2}\mathcal{L}_{\vec{n}} g_{\mu\nu} = \frac{b'}{b} g_{\mu\nu}. \tag{36}$$

If the brane stands at $r = r_1$ relative to the left bulk, then (35) and (36) give

$$\widetilde{(\frac{b'}{b})} = x \left(-\epsilon b^{-2} + \frac{\lambda}{3} \right), \tag{37}$$

which yields, for positive curvature brane,

$$\lambda = 3 \left[\left(l_1 \sinh(\frac{r_1}{l_1}) \right)^{-2} + \frac{1}{x} \left(\frac{1}{l_1} \coth(r_1/l_1) + \frac{1}{l_2} \coth(r_2/l_2) \right) \right], \tag{38}$$

for negative curvature brane,

$$\lambda = 3 \left[- \left(l_1 \cosh(\frac{r_1}{l_1}) \right)^{-2} + \frac{1}{x} \left(\frac{1}{l_1} \tanh(r_1/l_1) + \frac{1}{l_2} \tanh(r_2/l_2) \right) \right], \tag{39}$$

and for Ricci flat brane

$$\lambda = 3\frac{1}{x} \left[\frac{1}{l_1} + \frac{1}{l_2} \right]. \tag{40}$$

Under these conditions: 1. the brane Lagrangian (31) does not contain induced gravity term R; 2. the instanton is mirror symmetric; and 3. the brane in the instanton is a positive curvature brane the junction condition (37) degenerates to the condition in [3]. In fig. 2 we plot the warping factor $b(r)$ across the bulk. For Ricci flat and negative cases the qualitative properties of the warping factors is the same as positive case.

IV. INSTANTON ACTION

In spirits of sum-over-history formulism path integral extends over all paths-differentiable and non-differentiable. we know that the measure is concentrated on non-differentiable paths

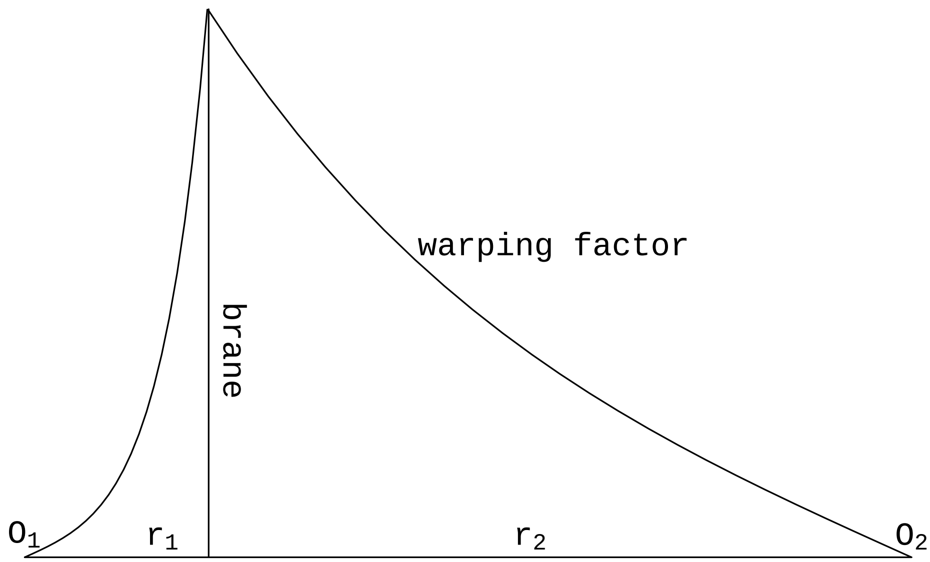

FIG. 2: Warping factor across the two bulk.

In this figure $l_2 = 4l_1$. One can see from the figure that the warping factor is not smooth at the position where brane stands, which provides the energy momentum stress tensor on the brane.

in the ordinary path integral case. In flat space case we can smooth the non-differentiable path by general analytical techniques. On the other hand because of the singularity theorem any reasonable spacetime must consist of one singularity at least. The discovery of cosmological constant, via breaking energy conditions, helps to escape the singularity theorem. But even in the case singular spacetime we can define the gravitational action in a meaningful way. In the case of black holes that the path integral should be taken over Euclidean, that is, positive definite metrics. This means that the singularities of black holes, like the Schwarzschild solution, do not appear on the Euclidean metrics which do not go inside the horizon. Instead the horizon is like the origin of polar coordinates. The action of the Euclidean metric is therefore well defined. Based on these considerations we permit singular paths, such as bulk which is Einstein space contain black strings. In the following we calculate the actions of various paths. As the first step we omit the induced gravity term in brane action (30) for a while . So the Euclidean action of the instanton

$$S_E = \frac{1}{16\pi G_5}\left(\int d^4x\sqrt{\det g_E}(4\overline{K}) + \sum_{L,R}\int d^5x\sqrt{\det({}^{(5)}g_E)}(2^{(5)}\Lambda - {}^{(5)}R)\right) + \frac{1}{16\pi G}\int d^4x\sqrt{\det g_E}(2\lambda - R) \tag{41}$$

becomes

$$S_E = \frac{1}{16\pi G_5}\left(\int d^4x\sqrt{\det g_E}(4\overline{K}) + \sum_{L,R}\int d^5x\sqrt{\det({}^{(5)}g_E)}(2^{(5)}\Lambda - {}^{(5)}R)\right)$$
$$+\frac{1}{16\pi G}\int d^4x\sqrt{\det g_E}(2\lambda). \tag{42}$$

In case of mirror symmetric instanton $K^+ = -K^-$, $\overline{K} = 0$ then GH boundary term of the bulk vanishes. But in case of asymmetric instanton it is necessary to include such a term in the action. Using the junction condition (38) , the action of positive curvature brane in the negative curvature Einstein bulk,

$$S_E^+ = \frac{V^+}{2\pi G_5}W^+(l_1, l_2, r_1, r_2), \tag{43}$$

where

$$V^+ = \int d^4x\sqrt{\det h_E}\ , \tag{44}$$

h_E takes the positive curvature metric given in (2), and thus V^+ is dimensionless and

$$\begin{aligned} W^+(l_1, l_2, r_1, r_2) = & - \tfrac{l_1^4}{4l_2}\coth(r_2/l_2)\sinh^4(r_1/l_1) + \tfrac{3}{8}(l_1^2 r_1 + l_2^2 r_2) \\ & - \tfrac{1}{4}l_1^3\sinh(2r_1/l_1) + \tfrac{1}{32}l_1^3\sinh(4r_1/l_1) \\ & - \tfrac{3}{16}l_2^3\sinh(2r_2/l_2). \end{aligned} \tag{45}$$

By using (24) one can eliminates a parameter, for example r_2, from (45), then we obtain

$$\begin{aligned} W^+(l_1, l_2, r_1) = & \tfrac{1}{32}[12l_1^2 r_1 + 12l_2^3 arcsh(\tfrac{l_1}{l_2}\sinh(r_1/l_1)) \\ + & 16l_1^3\sinh^3(r_1/l_1)\sqrt{1 + (\tfrac{l_1}{l_2}\sinh(r_1/l_1))^2} - 8l_1^3\sinh(2r_1/l_1) + l_1^3\sinh(4r_1/l_1) \\ - & 8l_2^3\sinh(2arcsh(\tfrac{l_1}{l_2}\sinh(r_1/l_1))) + l_2^3\sinh(4arcsh(\tfrac{l_1}{l_2}\sinh(r_1/l_1)))]. \end{aligned} \tag{46}$$

One can check the result in [3] is a special case of our result (46) under the mirror symmetry condition $l_1 = l_2$.

Just by the same method we derive the action of instanton with negative curvature brane,

$$S_E^- = \frac{V^-}{2\pi G_5}W^-(l_1, l_2, r_1), \tag{47}$$

where

$$V^- = \int d^4x\sqrt{\det h_E}\ , \tag{48}$$

h_E takes the negative curvature metric given in (2), V^- is dimensionless and

$$\begin{aligned} W^+(l_1,l_2,r_1) = & -\ \tfrac{1}{2}l_1^3\cosh^3(r_1/l_1)(1+\tfrac{l_1}{l_2}\cosh(r_1/l_1))\sqrt{1-\tfrac{2l_2}{l_2+l_1\cosh(r_1/l_1)}} \\ & +\ \tfrac{1}{32}[l_1^2(12r_1+8l_1\sinh(2r_1/l_1)+l_1\sinh(4r_1/l_1))+l_2^3(12arcsh(\tfrac{l_1}{l_2}\cosh(r_1/l_1)) \\ & +\ 8\sinh(2arcch(\tfrac{l_1}{l_2}\cosh(r_1/l_1)))+\sinh(4arcch(\tfrac{l_1}{l_2}\cosh(r_1/l_1))))]. \end{aligned} \tag{49}$$

The action of the instanton with Ricci flat brane can be obtained by analogy ,

$$S_E^{flat} = \frac{V^{flat}}{2\pi G_5} W^{flat}(l_1,l_2,r_1), \tag{50}$$

where

$$V^{flat} = \int d^4x\sqrt{\det h_E}\ , \tag{51}$$

h_E takes the Ricci flat metric given in (2), V^{flat} is also dimensionless and

$$W^{flat}(l_1,l_2,r_1) = -\frac{1}{4}\left[l_1^3(1-e^{4r_1/l_1})+(1+e^{4r_1/l_1})l_2^3\right]. \tag{52}$$

We know some Euclidean spaces whose 4-volumes are well defined. First for positive curvature Einstein branes there are two examples whose volumes we can calculate—de Sitter space and Nariai space. For a de Sitter brane

$$V^+(dS) = \frac{8}{3}\pi^2. \tag{53}$$

For a Nariai brane we have $m = \frac{1}{3\sqrt{3}}$ in (27) and the topology of the brane becomes $S^2 \times S^2$. Hereby

$$V^+(Na) = \frac{16}{9}\pi^2. \tag{54}$$

Then for Ricci flat Einstein branes we also presents some examples. One knows the volume of RS Euclidean (flat) brane is divergent without proper identity. But as we discussed above one can introduce instantons containing singularities. We calculate the actions of 4d Schwarzschild and Kerr metric here. Under these conditions we have to add a GH boundary term of the brane (boundary of boundary) in action (42). Considering this term , (52) becomes

$$S_E^{flat} = \frac{V^{flat}}{2\pi G_5} W^{flat}(l_1,l_2,r_1) + \frac{1}{8\pi G}\int d^3x I, \tag{55}$$

where I is the trace of the second fundamental form of the boundary of the brane. For Schwarzschild solution the extra part of the action is $4\pi G m^2$, therefore

$$S_E^{flat} = \frac{V^{flat}}{2\pi G_5} W^{flat}(l_1,l_2,r_1) + 4\pi G m^2. \tag{56}$$

Here m is the mass of the black hole. Note that the dimension m is [mass] in the above equation which is different from m in (27), where m is dimensionless. One knows that for Kerr solution the extra part of the action is [17]

$$2\pi m \frac{r_+^2 + J^2 m^{-2}}{r_+ - r_-},$$

where m is the black hole mass, r_+ is the radius of outer horizon, r_- is the inner horizon and J is the angular momentum of the black hole. There is another point to explain. One knows the schwarzschild black hole on the brane is an extensive object which is a black string in the 5d bulk. We only obtain the boundary term on brane, but how does the boundary of the string act off the brane? Generally speaking temperature makes no sense on an Einstein manifold with negative curvature. In fact the total actions of the bulk have been included in the first term in (55). However we still do not find the 4-volume of the brane appearing in (55) and then the concrete value of the action S_E^{flat} is left open. Nor do we know the 4-volume of the negative curvature brane without any compactifications "by hand". Whereas in a sense one can compare the actions of per unit 4-volume all the same. We draw actions per unit 4-volume W^+, W^{flat}, W^- in figs. 4 and 5, where the two instantons are in different asymmetric degrees. In order to contrast with symmetric case we also draw W^+, W^{flat}, W^- of mirror symmetric instantons in fig. 3.

Generally for mirror symmetric instanton the whole instanton is fixed if we fix the bulk on one side of the brane, that is, the bulk on the other side can be obtained by reflection and the energy momentum tensor on the brane is just the difference between the second forms along two sides of the brane. However for the asymmetric case fixing one half of the bulk is not enough to fix the whole instanton. We have many choices of other halves of bulk and the corresponding branes. It is interesting to study the actions of such a sequence of instantons. We present our results in fig. 6, 7 and 8.

Now we turn to the action with an induced Ricci term (41). Integrate it straightforwardly

$$S_E = -\left[\frac{1}{12\pi G_5}\sum_{L,R}\int dr b^4(r)^{(5)}\Lambda + \frac{\lambda b^4(r_1)}{24\pi G} + \frac{b^6(r_1)\epsilon}{4\pi G}\right] V \ , \tag{57}$$

where ϵ is defined by (9), V is the volume of the 4-Euclidean-Einstein brane with the metric $h_E = ds_4^2$ in (2) and V is a dimensionless number. The final integration in (57) is fairly easy while the final result is rather messy. But paying our attention to (33) and (34) we see that on an Einstein brane the effect of the induced Ricci tensor acts as a cosmological constant on

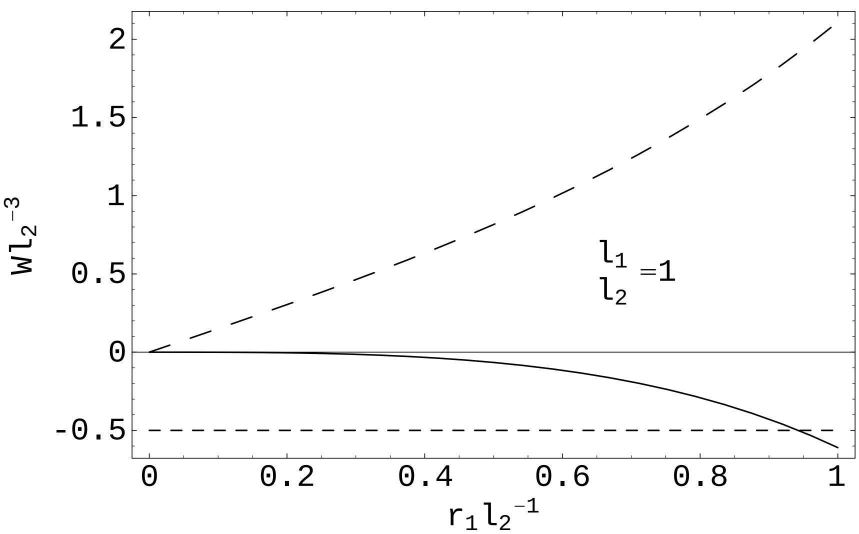

FIG. 3: Actions of per unit 4-volume of instantons with mirror symmetry. The solid curve denotes W^-/l_2^3, W^-/l_2^3 dwells on the long dashing curve and W^{flat}/l_2^3 resides on the short dashing curve.

the brane. We find the final result of (57) behaves similar to the case of the action without induced term (42) so it sheds no more light on our understanding of this asymmetric brean creation problem.

V. CONCLUSIONS AND DISCUSSIONS

We present a quantum cosmological scenario of brane world creation with an induced gravity term on the brane. In this scenario a brane is created together with bulk from nothing. The quantum creation is described by the brane instanton—a positive, negative or flat Einstein 4-manifold, which separates two asymmetric patches of negative curvature Einstein 5-manifolds. We study all three classes of branes residing in a negative curvature Einstein manifold and find they dwell in different positions, up to a boost isometry one another for different classes. All the branes in the same classes can be written in standard form simply by a rescaling of *radial* coordinate. Then we analyze the junction condition of brane with asymmetric bulk in induced gravity frame.

In Euclidean quantum gravity formulism, which is the application of quantum path integral formulism in gravity theory, we have

$$p \propto e^{-2S_E}, \tag{58}$$

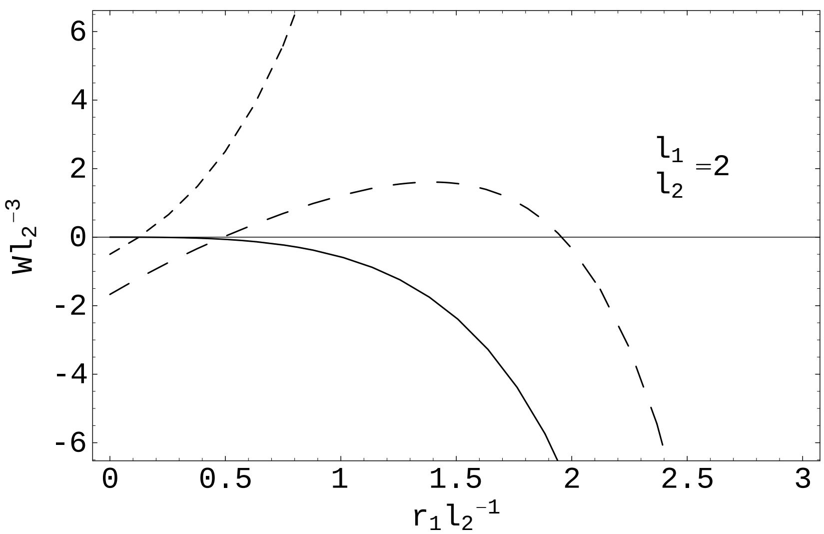

FIG. 4: Actions per unit 4-volume of instantons without mirror symmetry, in which $\frac{l_1}{l_2} = 2$, $l_2 = 1$. The solid curve denotes W^-/l_2^3, W^-/l_2^3 dwells on the long dashing curve and W^{flat}/l_2^3 resides on the short dashing curve.

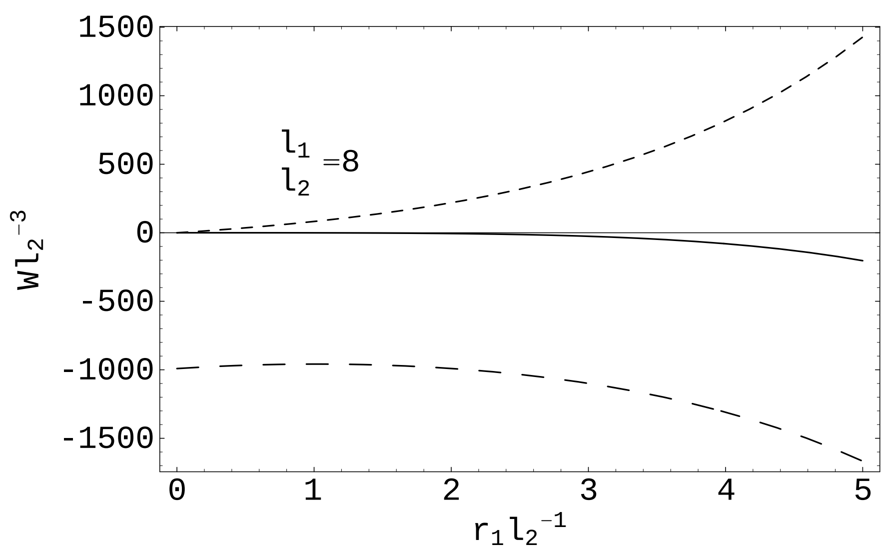

FIG. 5: Actions per unit 4-volume of instantons without mirror symmetry, in which $\frac{l_1}{l_2} = 8$, $l_2 = 1$. The solid curve denotes W^-/l_2^3, W^-/l_2^3 dwells on the long dashing curve and W^{flat}/l_2^3 resides on the short dashing curve.

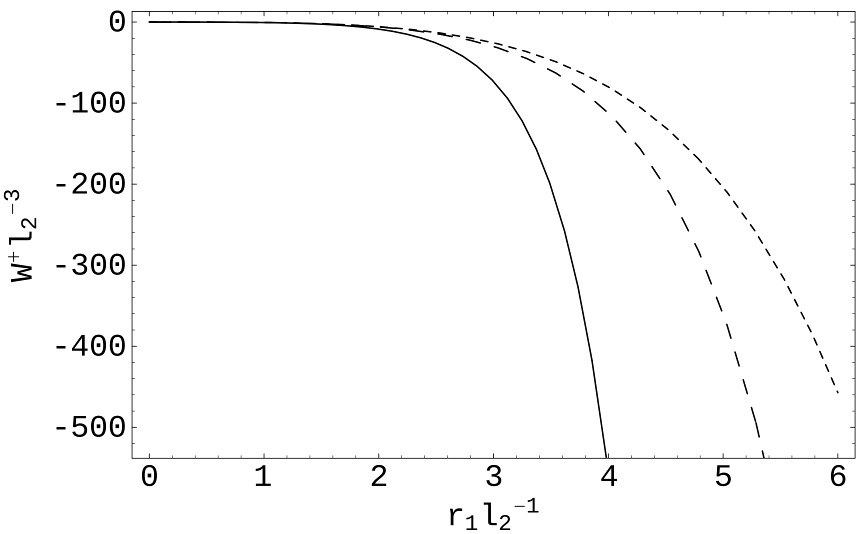

FIG. 6: Actions per 4-volume of positive brane instantons with a fixed half of bulk and a variable half of bulk. In this figure the right bulk with $l_2 = 1$ is fixed and the left bulk with $l_1/l_2 = 1$, $l_1/l_2 = 4$, $l_1/l_2 = 8$ respectively. The solid line denotes the left bulk of $l_1/l_2 = 1$, $l_1/l_2 = 4$ dwells on the long dashing line and the left bulk of $l_1/l_2 = 8$ resides on the short dashing line. As we expected the absolute value of the action decreases when r_1 increases.

where p is the probability associated to the path. So the action of an instanton may offer clues of the creation probabilities of the instantons. As an example from (53), (54) , (43), and (58) one can immediately say the de Sitter brane is more possibly created than Nariai brane. We investigate in detail the Euclidean action of three canonical types of instantons. We find that GH boundary term should be considered in the asymmetric case. We provide the analytical forms of the instanton by three parameters—the characteristic lengthes of the bulk on the left ,right and the position of the brane in instanton. We find for most brane metrics we are not so fortunate to obtain the definite action of an instanton. The total actions of open and flat brane are ill defined without proper identifyings put "by hand", as shown in section IV. Even in the case of positive brane, for instance simple as Schwarzschild-de Sitter metric, we do not know its Euclidean action very well because it is a non equilibrium system, i. e., its temperature is not definite. So we compared the actions of the three types of instantons per unit 4-volume. We also present the actions of instantons with a brane gluing a fixed bulk but different other bulk. All the three canonical types of branes are studied.

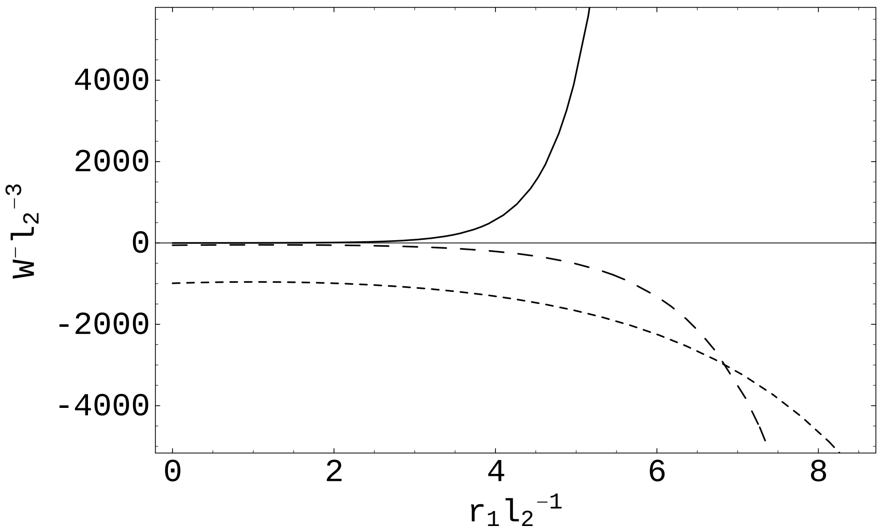

FIG. 7: Actions per 4-volume of positive brane instantons with a fixed half of bulk and a variable half of bulk. In this figure the right bulk with $l_2 = 1$ is fixed and the left bulk with $l_1/l_2 = 1$, $l_1/l_2 = 4$, $l_1/l_2 = 8$ respectively. One finds properties interestingly—the symmetric instanton always gets the biggest action (the least creation probability, if probability here is well defined). When r_1 is small the instanton $l_1/l_2 = 4$ get the least action but with increasing of r_1 the most the action asymmetric instanton $l_1/l_2 = 8$ soon becomes the smallest among the three.

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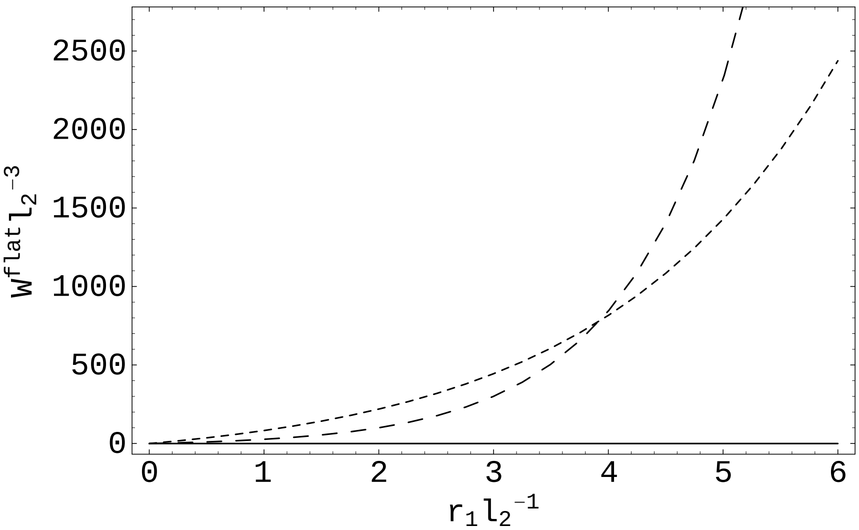

FIG. 8: Actions per 4-volume of positive brane instantons with a fixed half of bulk and a variable half of bulk. In this figure the right bulk with $l_2 = 1$ is fixed and the left bulk with $l_1/l_2 = 1$, $l_1/l_2 = 4$, $l_1/l_2 = 8$ respectively. The convention follows the above figure. From (52) we know W^f is exactly a constant in case of mirror symmetric instanton, as shown in the figure, and it always gets the least value.

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