QCD Sum Rules for Masses of Excited Heavy Mesons

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Abstract

The masses of excited heavy mesons are studied with sum rules in the heavy quark effective theory. A set of interpolating currents creating (annihilating) excited heavy mesons with arbitrary spin and parity are proposed and their properties are discussed. Numerical results at the leading order of the $O(1/m_Q)$ expansion are obtained for the lowest doublets $(0^+; 1^+)$ and $(1^+; 2^+)$.

Properties of excited heavy mesons composed of a heavy quark and a light anti-quark are of interest for several reasons. Some of these excited states have been observed in experiments. They will be objects of further study in future experiments with B-factories. In particular, they are useful for tagging in CP experiments [1]. Theoretically, the relative simplicity of the dynamical condition makes it of interest for exploring the internal dynamics of systems containing a light quark.

The $1/m_Q$ expansions are useful for such systems (m_Q is the heavy quark mass). The heavy quark effective theory (HQET) [2] is a powerful tool. However, for obtaining some detailed predictions one needs to combine it with some non-perturbative methods. The spectra and decay widths of heavy meson excited states have been studied with the $1/m_Q$ expansion in the relativistic Bethe-Salpeter equations in [3,4]. They can also be studied with the QCD sum rules in HQET which have been used for ground states of heavy mesons[5,6].

As a first step, in the present article we shall use the QCD sum rules in HQET to obtain the masses of lowest heavy meson doublets $(0^+;1^+)$ and $(1^+;2^+)$. These states have been studied with the sum rules in the full QCD in the literature [7]. We shall see in the following that using the HQET has some advantage for such problems.

In [8] it was shown that if the interaction kernel contains only γ and I on the heavy quark line, solutions of the Bethe-Salpeter equation for heavy mesons of arbitrary spin and definite parity consist of two series of degenerate spin doublets in the infinity quark mass limit $m_Q \to 1$. The wave functions $\phi_{jP\,i}$, where j, P are spin and parity and $i = 1;2$ for these two series of states, have the following most general form

$$\phi_{j;(1)^{j+1};1}(x;u) = e^{im_B v\cdot x}\sqrt{\frac{2j+1}{2j+2}}\frac{1+\not{v}}{2}\gamma_5\,\phi_1\cdots_j(-i)^j\frac{\partial}{\partial u_{t_2}}\cdots\frac{\partial}{\partial u_{t_j}}$$
$$(\frac{\partial}{\partial u_{t_1}}-\frac{j}{2j+1}\gamma_{t_1}\not{\frac{\partial}{\partial u_t}})(\varphi_{1j}+i\frac{\not{\partial}}{\partial u_t}\varphi_{2j}); \qquad (1a)$$

$$\phi_{j+1;(1)^{j+1};1}(x;u) = e^{im_B v\cdot x}\frac{1}{2}\frac{1+\not{v}}{2}\,\phi_1\cdots_{j+1}\gamma_{t_1}(-i)^j\frac{\partial}{\partial u_{t_2}}\cdots\frac{\partial}{\partial u_{t_{j+1}}}$$
$$(\varphi_{1j}+i\frac{\not{\partial}}{\partial u_t}\varphi_{2j}); \qquad (1b)$$

and

$$\phi_{j+1;(1)^j;2}(x;u) = e^{im_B v\cdot x}\frac{1}{2}\frac{1+\not{v}}{2}\gamma_5\,\phi_1\cdots_{j+1}\gamma_{t_1}(-i)^j\frac{\partial}{\partial u_{t_2}}\cdots\frac{\partial}{\partial u_{t_{j+1}}}$$
$$(\varphi_{1j}+i\frac{\not{\partial}}{\partial u_t}\varphi_{2j}); \qquad (2a)$$

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$$\chi_{j;\mu_1\cdots\mu_{2j+2}}(x;u) = e^{im_B v \cdot x} \sqrt{\frac{2j+1}{2j+2}} \frac{1+\rlap{/}v}{2} \epsilon_{\mu_1\cdots\mu_j} (-i)^j \frac{\partial}{\partial u_{t\,\mu_2}} \cdots \frac{\partial}{\partial u_{t\,\mu_j}}$$

$$(\frac{\partial}{\partial u_{t\,\mu_1}} - \frac{j}{2j+1} \gamma_t^{\nu} \frac{\partial}{\partial u_t^{\nu}})(\phi_{1j} + i\gamma \cdot \frac{\partial}{\partial u_t}\phi_{2j}) : \tag{2b}$$

In the above formulas, m_B is the meson mass, x is the coordinate of the heavy quark, u is the coordinate of the light anti-quark relative to the heavy quark, v is the velocity of the heavy meson, $u_l = v \cdot u$, $u_t = u - u_l v$, $\gamma_t = \gamma - \rlap{/}v v$, $\epsilon_{\mu_1\cdots\mu_j}$ is the transverse, symmetric and traceless polarization tensor. Finally, ϕ_i and φ_i are invariant functions of u_t^2 and u_l.

The assumptions used to obtain these results can be slightly generalized compared to that explicitly written down in [8]. We only need to assume that in the $m_Q \to \infty$ limit the kernel is of the form

$$G(P;p;p') = I \otimes \Omega\, U_S(P;p;p') + \gamma \otimes \Gamma\, U_V(P;p;p') \tag{3}$$

where P is the total momentum, p and p' are the relative momenta of the quarks in the initial and final state respectively, Ω and Γ are arbitrary functions of momenta and matrices for the light quark which transform as a scalar and a vector respectively, U_S and U_V are arbitrary invariant functions of momenta. Instead of the free propagator, the general form can be used for quark propagators appearing in the B–S equation. The calculations leading to above results only need to be slightly changed for this general case. The restriction to I and γ in (3) for the heavy quark is equivalent to that the interaction is independent of the heavy quark spin in the limit $m_Q \to \infty$. This is satisfied by QCD.[1] Besides this, the interaction kernel is most general. The B–S wave function is not gauge invariant so that functions ϕ_i and φ_i are gauge dependent. However, the general form of (1) and (2) is the same for any gauge. Physically, each doublet has a definite value for the total spin of the light component. Except for the states 0^- and 0^+, for each state in the series (1) there is a state with the same j, P in the series (2). They correspond to different values of $j_l = j + \frac{1}{2}$ and $j - \frac{1}{2}$. In the following we shall switch between the indices i and j_l. This pattern is in agreement with the heavy quark symmetry. The above results are model independent consequences of the heavy quark symmetry. The B–S equation,

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[1] In QCD the elementary interaction vertex of the quark with the gluon is γ, the heavy quark propagator is $\frac{1}{v \cdot k} \frac{1+\rlap{/}v}{2}$ in the $m_Q \to \infty$ limit. Using $\frac{1+\rlap{/}v}{2} \gamma \frac{1+\rlap{/}v}{2} = v \frac{1+\rlap{/}v}{2}$, the contribution of any Feynman diagram to the B–S kernel can be reduced to the form (3) after integrating out the internal momentum of the diagram.

being Lorentz covariant, only serves as a convenient tool for nding the form of the wave function of the state with de nite values for the angular momenta j, j_l and the parity P. For related results, see [9,10]. The authors of these latter works used directly the transformation properties of the wave function of a composite system with the angular momenta j and j_l. [10] contains results similar to ours, though the author did not deal exactly with the B-S wave function. This again implies that our results on the forms of the B-S wave functions in the $m_Q \to 1$ limit are model independent.

The relations between B-S wave functions and the matrix elements of the currents make the above results useful for constructing the appropriate interpolating currents and deriving their properties. The wave function ψ_{jP} in (1) and (2) can be written in the form

$$\psi_{j;P;i} = e^{im_B v \cdot x}\frac{1+\rlap{/}{v}}{2}\gamma_1 \cdots \gamma_j \Phi_{j;P;i}^{\mu_1 \cdots \mu_j}\left[(\frac{\partial}{\partial u_t})f_1(u_t^2;u_l) + i\frac{\partial\!\!\!/}{\partial u_t}f_2(u_t^2;u_l)\right]: \qquad (4)$$

We propose to use

$$J_{j;P;i}^{Y, \mu_1 \cdots \mu_j}(x) = \bar{h}_v(x)\Gamma_{j;P;i}^{f, \mu_1 \cdots \mu_j g}(D_{x_t})q(x) ; \qquad (5a)$$

or

$$J_{j;P;i}^{0\,Y, \mu_1 \cdots \mu_j}(x) = \bar{h}_v(x)\Gamma_{j;P;i}^{f, \mu_1 \cdots \mu_j g}(D_{x_t})(-i)\rlap{/}{D}_{x_t}q(x) ; \qquad (5b)$$

as the interpolating current which creates the corresponding heavy meson, where $h_v(x) = e^{im_B v \cdot x}\frac{1+\rlap{/}{v}}{2}Q(x)$ is the heavy quark eld in HQET and $Q(x)$ is that in full QCD, D_{x_t} is the covariant derivative $\frac{\partial}{\partial x_t} - igA_t(x)$ and

$$\Gamma^{f, \mu_1 \cdots \mu_j g}(D_{x_t}) = \text{Symmetrize}\left[\gamma^{\mu_1} \cdots \gamma^{\mu_j}(D_{x_t}) - \frac{1}{3}g_t^{\mu_1 \mu_2}g^{t}_{\mu_1' \mu_2'}\gamma^{\mu_1' \mu_2' \mu_3 \cdots \mu_j}(D_{x_t})\right]$$

with $g_t^{\mu_1 \mu_2} = g^{\mu_1 \mu_2} - v^{\mu_1}v^{\mu_2}$.

The currents hermitian conjugate to (5a) and (5b)

$$\bar{J}_{j;P;i}^{\mu_1 \cdots \mu_j}(x) = \bar{q}(x)\bar{\Gamma}_{j;P;i}^{f, \mu_1 \cdots \mu_j g}(D_{x_t})h_{v(x)} ; \qquad (6a)$$

$$\bar{J}_{j;P;i}^{0, \mu_1 \cdots \mu_j}(x) = \bar{q}(x)(i\rlap{/}{D}_{x_t})\bar{\Gamma}_{j;P;i}^{f, \mu_1 \cdots \mu_j g}(D_{x_t})h_{v(x)} ; \qquad (6b)$$

where $\bar{\Gamma} = \gamma^0 \Gamma^Y \gamma^0$, annihilate the same meson. The currents de ned in (5) and (6) have nice properties. Let $jj;P;ii$ be the heavy meson state with the quantum numbers j, P, i in the $m_Q \to 1$ limit. We have

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$$\langle 0 | J_{j;P;i}^{1\ \ j}(0) | j^0;P^0;i^0 \rangle = f_{P\ j_l}\ _{jj^0}\ _{PP^0}\ _{ii^0}\ ^1\ ^j\ ; \tag{7}$$

where $f_{jP\ i} = f_{P\ j_l}$ has the same value for the two states in the same doublet. The formula (7) can be verified as the following. Without loss of generality we can choose a gauge satisfying $A(x_0) = 0$. In such a gauge, it follows from the definition of the B–S wave function $_{j;P;i}(x;u) = \langle 0 | T(Q(x)q(x+u)) | j;P;i \rangle$ that for a current in (6a)

$$\langle 0 | J_{j;P;i}^{1\ \ j}(x_0) | j^0;P^0;i^0 \rangle = \mathrm{tr}\ _{j^0;P^0;i^0}(0;u)\ _{j;P;i}^{f\ 1\ \ jg}(@_{u_t})\Big|_{u=0}\ e^{i\ v\ x_0}$$

$$= \mathrm{tr}\ _{j;P;i}^{f\ 1\ \ jg}(@_{u_t})\frac{1 + 6}{2}\ _{j^0;P^0;i^0}^{1\ \ j^0}(@_{u_t})\Big(f_1(u_t^2;u_l) + i@_{u_t}f_2(u_t^2;u_l)\Big)\Big|_{u=0}\ _1\ _{j^0}e^{i\ v\ x_0}\ ; \tag{8}$$

where $= m_h\ m_Q$. Using the forms of $\ $ in (1) and (2) and the relations of the type

$$\frac{@}{@u_{t\ 1}}\quad \frac{@}{@u_{t\ 2l+1}}f(u_t^2;u_l)\Big|_{t=0} = 0\ ; \tag{9a}$$

$$S\left(\frac{@}{@u_{t\ 1}}\quad \frac{@}{@u_{t\ j}}\right)\left(\frac{@}{@u_{t\ 1}}\quad \frac{@}{@u_{t\ j}}\right)f(u_t^2;u_l)\Big|_{t=0}$$

$$= \frac{@^j f}{(@u_t^2)^j}\ 2^j\ j!\ S\ g_t^{1\ 1}\quad _t^{j\ j}\ _{jj^0}\Big|_{t=0}\ ; \tag{9b}$$

where S denotes symmetrization and subtracting the trace terms in the sets $(_1\quad _j)$ and $(_1\quad _{j^0})$, (7) can be obtained from (8). Only the term f_1 contributes to $f_{P\ j_l}$. If instead of J we use J^0 in (6b), the calculation is almost the same, except that now only the f_2 term contributes to $f_{P\ j_l}$.

Using the equation of motion for the light quark $i\hat{6}\ q = 0$ and that for the heavy quark in HQET $iv\ D\ h = 0$, one can obtain the relation

$$f_{jP\ i}^0 = (\ 1)^{i+1}\ _{jP\ i}f_{jP\ i} \tag{10}$$

between the coupling constants for the current with the extra factor $i\hat{6}$ and that without it.

Similarly, calculated with the quark and gluon fields in the $m_Q\ !\ 1$ limit the correlation functions of $J_{j;P;i}$ and $J_{j^0;P^0;i^0}^Y$ satisfy

$$i\langle 0 | T\left(J_{j;P;i}^{1\ \ j}(x)J_{j^0;P^0;i^0}^{Y\ 1\ \ j^0}(0)\right) | 0 \rangle = _{jj^0}\ _{PP^0}\ _{ii^0}(\ 1)^j\ S\ g_t^{1\ 1}\quad _t^{j\ j}$$

$$\int dt\ (x\quad vt)\ _{P;j_l}(x)\ ; \tag{11}$$

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where $\zeta_{P;j_l}$ is the same function for the two states in the same doublet. For verifying (11) we use the Schwinger gauge

$$x \cdot A = 0: \tag{12}$$

The heavy quark propagator in HQET is

$$-i\langle 0|T\{h_v(x)\bar{h}_v(0)\}|0\rangle = \frac{1+\not{v}}{2}\int dt\,\delta^4(x-vt): \tag{13}$$

The leading order Lagrangian in HQET is $L_0 = i\bar{h}_v v\cdot D h_v$. Therefore in gauge (12) there is no gluon line terminating on the heavy quark line. We thus obtain

$$-i\langle 0|T\{J_{j;P,i}(x)J^{\nu}_{j^0;P^0,i^0}(0)\}|0\rangle = -i\int dt\,\delta^4(x-vt)\langle 0|T\left\{\mathrm{tr}_{(color)}\mathrm{tr}\left[\frac{1+\not{v}}{2}\right.\right.$$
$$\left.\left. \Gamma_{j^0;P^0,i^0}(D_{yt})q(y)\bar{q}(x)\Gamma_{j;P,i}(D_{xt})\right]\right\}\bigg|_{y=0,x=vt}: \tag{14}$$

Using the relation

$$\partial_{x_{t_1}}\cdots\partial_{x_{t_j}}\partial_{y_{t_1}}\cdots\partial_{y_{t_{j^0}}}\mathrm{tr}_{(color)}\left[q(y)\bar{q}(x)\exp\left(ig\int_y^x dz\cdot A\right)\right]\bigg|_{y=0,x=vt}$$
$$= \mathrm{tr}_{(color)}\left[D_{y_{t_1}}\cdots D_{y_{t_{j^0}}}q(y)\bar{q}(x)D_{x_{t_1}}\cdots D_{x_{t_j}}\right]\bigg|_{y=0,x=vt};$$

in the gauge (12), one can rewrite (14) in the form

$$-i\langle 0|T\{J_{j;P,i}(x)J^{\nu}_{j^0;P^0,i^0}(0)\}|0\rangle = -i\int dt\,\delta^4(x-vt)\mathrm{tr}\left[\Gamma_{j^0;P^0,i^0}\right.$$
$$\left.(\overleftarrow{\partial}_{y_t})S(y;x)\Gamma_{j;P,i}(\overrightarrow{\partial}_{x_t})\frac{1+\not{v}}{2}\right]\bigg|_{y=0,x=vt}; \tag{15}$$

where

$$S(y;x) = \mathrm{tr}_{(color)}\langle 0|T\left\{q(y)\bar{q}(x)\exp\left(ig\int_y^x dz\cdot A\right)\right\}|0\rangle: \tag{16}$$

From Lorentz and translation invariance for gauge invariant quantities, $S(y;x)$ has the general form

$$S(y;x)\frac{1+\not{v}}{2} = \left[S_1((y-x)_t^2;y_l-x_l) + i\not{\partial}_{x_t}S_2((y-x)_t^2;y_l-x_l)\right]\frac{1+\not{v}}{2}: \tag{17}$$

Substituting (17) into (15) brings it into a form almost identical to the r.h.s. of (8). The formula (11) then follows after similar calculations.

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(7) and (11) imply that two currents in the sets (5a) and (5b) with not identical values of j, P, i never mix in the $m_Q \to 1$ limit either in the heavy meson or in the quark-gluon level and heavy quark symmetry is explicit with these currents. This verifies that they are the appropriate interpolating currents for heavy meson states with definite j, P and the light quark momentum j_l which are conserved in QCD in $m_Q \to 1$ limit.

The properties for currents mentioned above are important for applications to QCD sum rules for excited heavy mesons. If one use other currents or study the sum rules in full QCD, there are in general contributions from two nearby poles corresponding to states of the same j, P. Their contributions may not be separated correctly. Furthermore, the mixing of such two states can be calculated within our formalism by introducing the $O(1/m_Q)$ terms in the Lagrangian in HQET.

For obtaining QCD sum rules in the HQET we study the correlator

$$\Pi_{j;P;i}^{\mu_1 \cdots \mu_j; \nu_1 \cdots \nu_j}(k) = i \int d^4x e^{ik \cdot x} \langle 0 | T \left(J_{j;P;i}^{\mu_1 \cdots \mu_j}(x) J_{j;P;i}^{\dagger \nu_1 \cdots \nu_j}(0) \right) | 0 \rangle : \tag{18}$$

From (8), it has the following form at the leading order

$$\Pi_{j;P;i}^{\mu_1 \cdots \mu_j; \nu_1 \cdots \nu_j}(k) = (-1)^j S \, g_t^{\mu_1 \nu_1} \cdots g_t^{\mu_j \nu_j} \, \Pi_{j;P;i}(\omega) ; \tag{19}$$

where $\omega = 2v \cdot k$ and $\Pi_{j;P;i}(\omega) = \Pi_{P;j_l}(\omega)$ is independent of j.

We shall confine us to the leading order terms in this article. The contribution to $\Pi_{P;j_l}(\omega)$ of the pole term on the hadron side is

$$\frac{f_{P;j_l}^2}{2\bar{\Lambda} - \omega} :$$

On the quark-gluon side of the sum rule, $\Pi_{P;j_l}(\omega)$ is calculated with the leading order Lagrangian L_0 in the HQET. The Feynman diagrams contributing to (18) are shown in Fig 1. For the sake of simplicity we consider the doublets $(0^+; 1^+)$ and $(1^+; 2^+)$ in the following.

According to (5a) and (5b), there are two possible choices for currents for the doublet $(0^+; 1^+)$, either

$$J_{0;+;2}^{\dagger} = \bar{h}_v q ; \tag{20}$$

$$J_{1;+;2}^{\dagger} = \bar{h}_v \gamma_5 \gamma_t^{\mu} q ; \tag{21}$$

or

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$$J_{0;+;2}^{0y} = \frac{1}{P=2} h_v (\ i) \not{D}_t q \ ; \tag{22}$$

$$J_{1;+;2}^{0y} = \frac{1}{P=2} h_v \ ^5 \ _t (\ i) \not{D}_t q : \tag{23}$$

Similarly, there are two possible choices for the currents for the doublet $(1^+;2^+)$. One is

$$J_{1;+;1}^{y} = \sqrt{\frac{3}{4}} h_v \ ^5 (\ i) \ D_t \quad \frac{1}{3} \ _t \not{D}_t \quad q \ ; \tag{24}$$

$$J_{2;+;1}^{y \ 1; \ 2} = \sqrt{\frac{1}{2}} h_v \frac{(\ i)}{2} (\ _t^1 D_t^2 + \ _t^2 D_t^1 \quad 2 g_t^{1 \ 2} \not{D}_t) q : \tag{25}$$

Another choice is obtained by adding a factor $i \not{D}_t$ to (24) and (25). Note that, without the last term in the bracket in (24) the current would couple also to the 1^+ state in the doublet $(0^+;1^+)$ even in the limit of in nite m_Q.

Usually, the currents with the least number of derivatives are used in the QCD sum rule approach. The sum rules with them have better convergence in the high energy region and often have better stability. However, there is a motivation for using the currents (22), (23) for the doublet $(0^+;1^+)$. In the non-relativistic quark model, which usually gives correct ordering of energy levels of hadron states, the doublets $(0^+;1^+)$ and $(1^+;2^+)$ are orbital p-wave states which correspond to one derivative in the space wave functions. In fact, as mentioned above, the coupling constant for the currents (22), (23) is proportional to $_2$ in (2) and that for the currents (24), (25) is proportional to $_1$ in (1) which are the large components in the non-relativistic approximation. Therefore, we shall consider both the currents (20), (21) and (22), (23) for the doublet $(0^+;1^+)$.

First we consider the doublet $(0^+;1^+)$. For the correlator

$$_{0;+;2} = i \int d^4 x e^{ik \cdot x} h0 \mid T \quad J_{0;+;2}^0 (x) J_{0;+;2}^{0y} (0) \quad j0 i \ ; \tag{26}$$

where $J_{0;+;2}^0$ is the current conjugate to (22), the sum rules after the Borel transformation is found to be

$$f^2 e^{2 \ =T} = \frac{3}{2^6 \ ^2} \int_0^{!_c} !^4 e^{!=T} d! \quad \frac{1}{2^4} m_0^2 h\bar{q}qi \ ; \tag{27}$$

$$= \frac{\dfrac{3}{2^3 \ ^2} \int_0^{!_c} !^5 e^{!=T} d!}{\dfrac{3}{4 \ ^2} \int_0^{!_c} !^4 e^{!=T} d! \quad m_0^2 h\bar{q}qi} : \tag{28}$$

In the above sum rule we have con ned us to terms of lowest order in perturbation and operators of dimension less than six.

The correlator of the 1^+ current $J^0_{1;+;2}$ (23) and its conjugate has the form

$$\Pi_{1;+;2} = i \int d^4x\, e^{ik\cdot x} \langle 0|T\{J^0_{1;+;2}(x)J^{0\gamma}_{1;+;2}(0)\}|0\rangle = -g_t\, \Pi_{1;+;2}(\omega):\tag{29}$$

We find for $\Pi_{1;+;2}(\omega)$ the identical sum rule as that for $\Pi_{0;+;2}(\omega)$, as it should be.

The corresponding formula when the current $J_{0;+;2}$ and $J_{1;+;2}$ in (20) and (21) are used instead of $J^0_{0;+;2}$ and $J^0_{1;+;2}$ are the following

$$f^2 e^{2\bar{\Lambda}=T} = \frac{3}{16\pi^2}\int_0^{\omega_c} \omega^2 e^{-\omega=T}\, d\omega + \frac{1}{2}\langle \bar{q}q\rangle - \frac{1}{8T^2}m_0^2\langle \bar{q}q\rangle;\tag{30}$$

$$\bar{\Lambda} = \frac{\dfrac{3}{2^4\pi^2}\int_0^{\omega_c}\omega^3 e^{-\omega=T}\, d\omega + \dfrac{m_0^2}{4T}\langle \bar{q}q\rangle}{\dfrac{3}{2^3\pi^2}\int_0^{\omega_c}\omega^2 e^{-\omega=T}\, d\omega + \langle \bar{q}q\rangle - \dfrac{1}{4T^2}m_0^2\langle \bar{q}q\rangle}\tag{31}$$

The correlators for the $(1^+;2^+)$ doublet have the form

$$\Pi^{\mu;\nu}_{1;+;1}(k) = -g_t\, \Pi_{+;j_l=3=2}(\omega);\tag{32}$$

$$\Pi^{\mu_1\mu_2;\nu_1\nu_2}_{2;+;1}(k) = \frac{1}{2}\left[g_t^{\mu_1\nu_1}g_t^{\mu_2\nu_2} + g_t^{\mu_1\nu_2}g_t^{\mu_2\nu_1} - \frac{2}{3}g_t^{\mu_1\mu_2}g_t^{\nu_1\nu_2}\right]\Pi_{+;j_l=3=2}(\omega):\tag{33}$$

When the currents (24) and (25) are used the sum rules for this doublet are found to be

$$f^2 e^{2\bar{\Lambda}=T} = \frac{1}{2^6\pi^2}\int_0^{\omega_c}\omega^4 e^{-\omega=T}\, d\omega - \frac{1}{12}m_0^2\langle \bar{q}q\rangle - \frac{1}{2^5}\langle h\frac{\alpha_s}{\pi}G^2\rangle iT;\tag{34}$$

$$\bar{\Lambda} = \frac{\dfrac{1}{2^5\pi^2}\int_0^{\omega_c}\omega^5 e^{-\omega=T}\, d\omega - \dfrac{1}{2^4}\langle h\frac{\alpha_s}{\pi}G^2\rangle iT^2}{\dfrac{1}{2^4\pi^2}\int_0^{\omega_c}\omega^4 e^{-\omega=T}\, d\omega - \dfrac{m_0^2}{3}\langle \bar{q}q\rangle - \dfrac{1}{2^3}\langle h\frac{\alpha_s}{\pi}G^2\rangle iT}:\tag{35}$$

Here the $\langle h\frac{\alpha_s}{\pi}G^2\rangle i$ term comes from the diagram (c) in Fig. 1 which vanishes for the doublet $(0^+;1^+)$.

From (28), (31) and (35) we obtain curves shown in Fig. 2a,b,c. For the first two cases stability windows exist when ω_c lies in the interval 2.4 to 2.9 GeV; for the last one ω_c lies in the interval 2.7 to 3.2 GeV. Imposing usual criterium for the upper and lower bounds of the Borel transform parameter T we found approximately the same stability window 0.7{0.9 GeV. The results for $\bar{\Lambda}$ for three cases are the following

$$\bar{\Lambda}(1=2^+) = 1.05 \pm 0.05 \text{ GeV};\tag{36}$$

for the doublet $(0^+;1^+)$ when the currents (20), (21) without the derivative is used.

$$(1=2^+) = 0.90 \quad 0.10 \quad \text{GeV};\tag{37}$$

for the same doublet when the currents (22), (23) with the derivative is used.

$$(3=2^+) = 0.95 \quad 0.10 \quad \text{GeV};\tag{38}$$

for the doublet $(1^+;2^+)$.

The predicted mass value for the doublet $(0^+;1^+)$ in (36) is somewhat different from that in (37). The value in (36) is in agreement with the result of [7] in which the same currents as (20) and (21) are used. (36) predicts a mass value of the doublet $(0^+;1^+)$ slightly higher than the mass value of the doublet $(1^+;2^+)$ predicted in (38). On the other hand, (37) gives a mass value for the doublet $(0^+;1^+)$ approximately equal or slightly lower than that for the doublet $(1^+;2^+)$ in (38). Though (36) and (37) can be considered as consistent within the uncertainties, (37) indicates that the QCD sum rule approach can incorporate a mass value for $(0^+;1^+)$ lower than that for $(1^+;2^+)$. This is in agreement with quark model predictions in [11] and [3]. By contrast, we have also derived the sum rule for the ground state doublet $(0\ ;1\ )$ with the currents with the extra derivative iD. We found that there is no acceptable stability window for this sum rule.

Finally, we would like to address the following points of this paper. The results from our previous work on B-S wave functions are used only for finding the appropriate currents. Once these currents are given, the calculations in QCD sum rules are independent of the consideration of the B-S equation. That the currents used are appropriate was checked by proving (11) with direct calculations in terms of quark-gluon fields in HQET without referring to the B-S wave functions. Assuming duality and inserting intermediate heavy meson states between J and J^y in (11), this equation also strongly suggests equation (7) which is the direct statement that the currents create pure states with definite values of j, P and j_l.

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Figure Captions

Fig. 1. Feynman diagrams contributing to the sum rule. There is no Fig (c) when the currents (20) (21) are used.

Fig. 2. Dependence of on the Borel parameter T for different values of the continuum threshold $!_c$. (a) (b) for $(0^+;1^+)$ doublet with the interpolating currents (22) (23) and (20) (21) used respectively. From top to bottom the curves correspond to (a) $!_c = 2.8;2.6;2.4$ GeV and (b) $!_c = 2.9;2.65;2.4$ GeV. (C) for doublet $(1^+;2^+)$. From top to bottom the curves correspond to $!_c = 3.1;2.9;2.7$ GeV respectively.

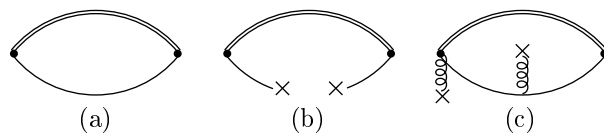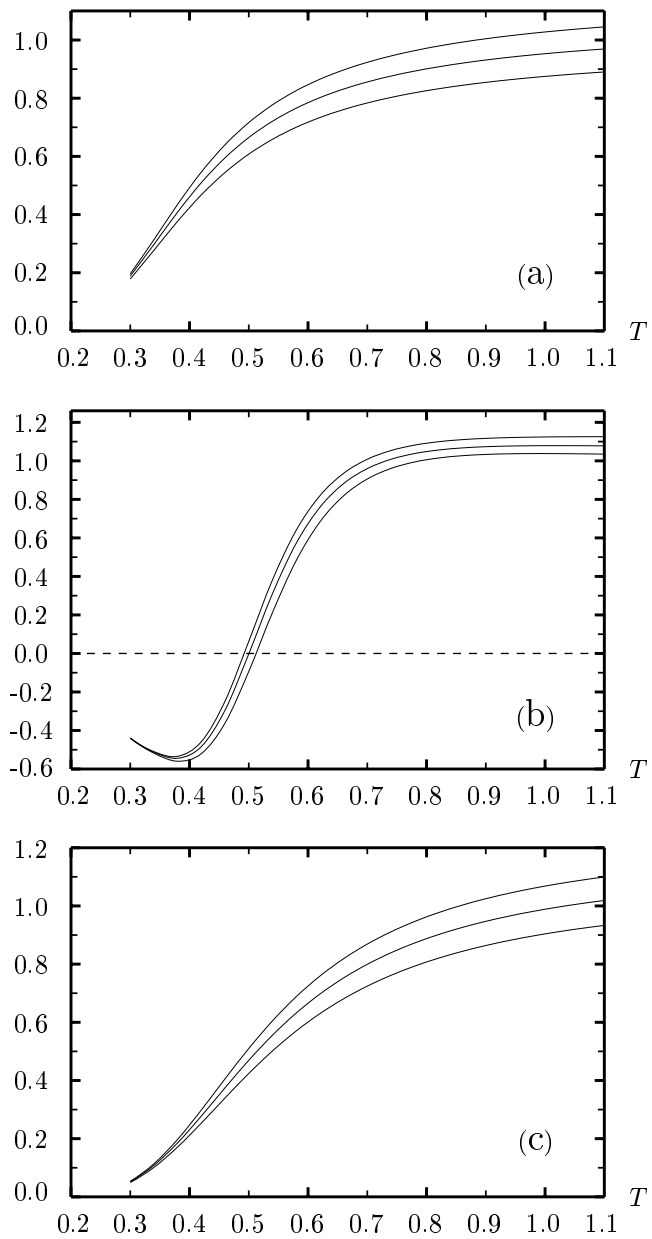**Fig 1**

Fig 2