Top quark three-body decays in $R$-violating MSSM

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Abstract

In the minimal supersymmetric standard model the R-parity violating interactions can trigger various exotic three-body decays for the top quark, which may be accessible at the LHC. In this work we examine the R-violating decays $t \to cX_1X_2$, which include the tree-level processes $t \to c\ell_i\ell_j^+ (\ell_i = e, \mu, \tau)$ and $t \to cd_i\bar{d}_j (d_i = d, s, b)$, as well as the loop-induced processes $t \to cgX (X = g, \gamma, Z, h)$. We find that the hereto weakly constrained R-violating couplings can render the decay branching ratios quite sizable, some of which already reach the sensitivity of the Tevatron collider and can be explored at the LHC with better sensitivity.

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I. INTRODUCTION

Top quark is one of the forefront topics in high energy physics. As the heaviest known elementary particle, top quark is speculated to be a window into the TeV-scale physics. The properties of the top quark have been being measured at the Tevatron collider and so far the Tevatron data are in agreement with the SM predictions. However, due to the small statistics of the Tevatron collider, the precision of current measurements of the top quark properties is not so good and hence there remains plenty of room for new physics in the top quark sector. The CERN Large Hadron Collider (LHC) will serve as a top quark factory and allow to scrutinize the top quark nature. The precise measurement of the top quark properties at the LHC may provide clues to new physics beyond the Standard Model (SM) [1].

Due to the large number of top pair samples at the LHC, various exotic decays of the top quark could be explored with high sensitivity. In the SM top quark dominantly decays into a $W$-boson plus a bottom quark. But in new physics models various exotic decays can open up and may reach the detectable level. For example, the FCNC two-body decays $t \to cX$ ($X = g, \gamma, Z, h$), which are extremely small in the SM [2], could be enhanced to the observable level in the minimal supersymmetry model (MSSM) [3, 4] and the technicolor models [5]. Also, some new decay modes may open up in new physics models, such as $t \to \tilde{\chi}^0_1 \tilde{t}$ in the MSSM [6]. Given the stringent lower bounds on the masses of new particles (like the top-squark $\tilde{t}$ and the lightest neutralino $\tilde{\chi}^0_1$) from current experiments, such two-body new decay modes are getting kinematically suppressed or forbidden. In this work we focus on the kinematically allowed three-body decays with all the final states being the SM particles.

It is well known that in the MSSM the $R$-parity, defined by $R = (-1)^{2S+3B+L}$ with spin $S$, baryon-number $B$ and lepton-number $L$, is often imposed on the Lagrangian to maintain the separate conservation of $B$ and $L$. But this conservation requirement is not dictated by any fundamental principle such as gauge invariance and renormalizability. Therefore, the phenomenology of $R$-parity violation has attracted much attention. For the effects of $R$-violating interactions in the top quark sector, we may have large FCNC two-body decays $t \to cX$ ($X = g, \gamma, Z, h$) and some exotic top productions at the LHC [7]. Note that the $R$-violating couplings can also induce various three-body decays for the top quark. In this
work we examine the three-body decays with all the final states being the SM particles, which include the tree-level processes \( t \to c \ell_i^- \ell_j^+ \) (\( \ell_i = e, \mu, \tau \)) and \( t \to cd_i \bar{d}_j \) (\( d_i = d, s, b \)), as well as the loop-induced processes \( t \to cgX \) (\( X = g, \gamma, Z, h \)). Although these three-body decays may be quite rare, they are still worth checking because the top decays can be soon scrutinized at the LHC. As will be shown by our study, the hereto weakly constrained R-violating couplings can make the decay branching ratios quite sizable, some of which already marginally reach the sensitivity of the Tevatron collider.

Note that the top quark three-body decays with one or two new (heavy) particles in the final states have been studied in the MSSM with or without R-parity [8–10] and in the general two-Higgs-doublet model [11]. In [9] the L-violating decay \( t \to c \ell_i^- \ell_j^+ \) was also studied. In our study we include it for completeness.

This work is structured as follows. In Sec. II, we recapitulate the R-parity violating couplings and present the calculations for top three-body decays at the LHC. In Sec. III, we show some numerical results for the branching ratios of these decays. In Sec. IV we draw our conclusion. The analytic expressions from the loop calculations are presented in the Appendix.

II. CALCULATION

The R-parity violating superpotential of the MSSM is given by [12]

\[
\frac{1}{2} \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \frac{1}{2} \lambda''_{ijk} U^c_i D^c_j D^c_k
\]  

where \( L_i(Q_i) \) and \( E^c_i(U^c_i, D^c_i) \) are respectively the doublet and singlet lepton (quark) chiral superfields, and \( i, j, k \) are generation indices. The terms of \( \lambda \) and \( \lambda' \) violate lepton number while the terms of \( \lambda'' \) violate baryon number. The non-observation of the proton decay imposes very strong constraints on the product of the \( L \)-violating and \( B \)-violating couplings [13]. Thus in our numerical calculation we will assume that only one type of these interactions (either \( L \)- or \( B \)-violating) exist. Since only \( \lambda' \) or \( \lambda'' \) couplings can induce the three-body top quark decays, we will drop the \( \lambda \) couplings in the following. In terms of the four-component Dirac notation, the Lagrangian of \( \lambda' \) and \( \lambda'' \) couplings is given by

\[
\mathcal{L}_{\lambda'} = -\frac{1}{2} \lambda'_{ijk} \left[ \bar{\nu}_i^j d_R^k d_L^i + \tilde{d}_R^j \bar{d}_L^i |\bar{\nu}_L^i| + \tilde{d}_R^j \nu^c_L |d_L^i| + \tilde{\nu}_L^i d_R^k u_R^i - \bar{\nu}_L^i \tilde{d}_R^k u_R^i - \tilde{\nu}_L^i \bar{d}_R^k \tilde{u}_R^i \right] + h.c.
\]

\[
\mathcal{L}_{\lambda''} = -\frac{1}{2} \lambda''_{ijk} \left[ \tilde{u}_R^j \bar{u}_R^k d_L^i + \tilde{d}_R^j \bar{u}_R^k d_L^i + \tilde{d}_R^j \bar{d}_R^k u_L^i \right] + h.c.
\]
The three-body decays $t \rightarrow c\ell_i^- \ell_j^+$ ($i$ and $j$ can be equal or not equal) can be induced at tree-level by the L-violating $\lambda'_{2ik} \lambda'_{j3k}$, as shown in Fig.1(a); while $t \rightarrow c\ell_i^- \ell_i^+$ can also be induce at loop-level by the B-violating $\lambda''_{2jk} \lambda''_{3jk}$, as shown in Fig.1(b) where the effective vertices $tc\gamma$ and $tcZ$ are similar to the effective vertex $tcg$ defined in Fig.3. The decays $t \rightarrow cd_i \bar{d}_j$ can be induced at tree-level either by the L-violating $\lambda'_{3ki} \lambda'_{k3i}$ or by the B-violating $\lambda''_{2ik} \lambda''_{3jk}$, as shown in Fig.1(c) and (d), respectively. The decay $t \rightarrow cg\gamma$ can be induced at loop-level by the L-violating $\lambda'_{2ik} \lambda'_{j3k}$ or the B-violating $\lambda''_{2jk} \lambda''_{3jk}$ as shown in Fig.2 with the effective vertex $tcg$ defined in Fig.3. The Feynman diagrams for $t \rightarrow cg\gamma, cgZ, cgh$ are similar to $t \rightarrow cgg$ and are not plotted here. The analytic expressions of the amplitudes for the loop-induced processes are lengthy and tedious. Here, as an example, we list the expressions for the L-violating loop contributions to the effective vertex $tcV$ in Appendix A. The corresponding B-violating contributions can be found in the third paper in [4].

![Feynman diagrams](image)

**FIG. 1:** Feynman diagrams: (a) $t \rightarrow c\ell_i^- \ell_j^+$ induced at tree-level by $\lambda'_{2ik} \lambda'_{j3k}$; (b) $t \rightarrow c\ell_i^- \ell_i^+$ induced at loop-level by $\lambda''_{2jk} \lambda''_{3jk}$ where the effective vertices $tc\gamma$ and $tcZ$ are similar to the effective vertex $tcg$ defined in Fig.3; (c-d) $t \rightarrow cd_i \bar{d}_j$ induced at tree-level by $\lambda'_{3ki} \lambda'_{k3i}$ and $\lambda''_{2ik} \lambda''_{3jk}$, respectively. In (a) the charged lepton $\ell_i^-$ and the charm quark can be replaced respectively by a neutrino $\nu_i$ and a strange quark to give the process $t \rightarrow s\nu_i \ell_j^+$. The current upper bounds for all R-parity violating couplings are summarized in [14]. Table I is a list of current limits for those couplings relevant to our study, taken from [14]. We see that the constraints are quite weak for the couplings $\lambda''_{2jk}$ and $\lambda''_{3jk}$, which induce the tree-level decays shown in Fig.1(d).

### III. NUMERICAL RESULTS AND DISCUSSIONS

In our calculation the top quark mass is taken as the new CDF value $m_t = 172$ GeV [15]. Other SM parameters are taken as [16] $m_Z = 91.19$ GeV, $m_W = 80.4$ GeV, $\sin^2 \theta_W = 0.2228$, ...
FIG. 2: Feynman diagrams for the decay $t \rightarrow cgg$ induced at loop-level by $\lambda_{i2k}^{'}\lambda_{i3k}^{''}$ or $\lambda_{2jk}^{''}\lambda_{3jk}^{''}$ with the effective vertex $tcg$ defined in Fig.3.

FIG. 3: Feynman diagrams for the effective vertex $tcg$ appeared in Fig.2, induced at loop-level by $\lambda_{i2k}^{'}\lambda_{i3k}^{''}$ or $\lambda_{2jk}^{''}\lambda_{3jk}^{''}$.

$\alpha_s(m_t) = 0.1095$ and $\alpha = 1/128$ [16]. The SUSY parameters involved in our calculations are the masses of squarks and sleptons as well as the R-parity violating couplings $\lambda_{i2k}^{'}\lambda_{i3k}^{''}$, $\lambda_{2jk}^{''}\lambda_{3jk}^{''}$, $\lambda_{i2k}^{''}$, $\lambda_{i3k}^{'}$, $\lambda_{2jk}^{'}$, and $\lambda_{3jk}^{'}$, whose upper bounds are listed in Table I. The strongest bound on squark mass is from the Tevatron experiment. For example, from the search for the inclusive production of squark and gluino in R-conserving minimal supergravity model with $A_0 = 0$, $\mu < 0$ and
TABLE I: Current upper limits on the R-parity violating couplings relevant to our study, taken from [14].

<table>
<thead>
<tr>
<th>couplings</th>
<th>bounds</th>
<th>sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda''<em>{212}$, $\lambda''</em>{213}$, $\lambda''_{223}$</td>
<td>1.25</td>
<td>perturbativity</td>
</tr>
<tr>
<td>$\lambda''<em>{312}$, $\lambda''</em>{313}$, $\lambda''_{323}$</td>
<td>$0.97 \times (m_{\tilde{d}_{kR}}/100 \text{ GeV})$</td>
<td>Z-decays</td>
</tr>
<tr>
<td>$\lambda'<em>{i1k}$, $\lambda'</em>{i2k}$</td>
<td>$0.11 \times \sqrt{m_{\tilde{d}_{kR}}/100 \text{ GeV}}$</td>
<td>$K^0 - \bar{K}^0$ mixing</td>
</tr>
<tr>
<td>$\lambda'_{k3k}$</td>
<td>$1.1 \times \sqrt{m_{\tilde{d}_{kR}}/100 \text{ GeV}}$</td>
<td>$B_d - \bar{B}_d$ mixing</td>
</tr>
<tr>
<td>$\lambda'<em>{i2k} \lambda''</em>{i3k}$</td>
<td>$0.09 \times (m_{\tilde{d}_{kR}}/100 \text{ GeV})^2$</td>
<td>$B \to K\gamma$</td>
</tr>
</tbody>
</table>

$tan \beta = 5$, the CDF gives a bound of 392 GeV at the 95 % C.L. [17] for degenerate gluinos and squarks. However, this bound may be not applicable to the R-violating scenario because the SUSY signal in case of R-violation is very different from the R-conserving case. The most robust bounds on sparticle masses come from the LEP results, which give a bound of about 100 GeV on squark or slepton mass [18]. In our numerical calculations we assume the presence of the minimal number of R-violating couplings, i.e., for each process only the two relevant couplings (not summed over the family indices) are assumed to be present.

In Fig. 4 we show the branching ratios of the three-body decays as a function of squark or slepton mass. In this figure the product of the two $\lambda''$ ($\lambda'$) couplings involved in each decay is fixed as 1.2 (0.1), which are approximately the maximal values shown in Table I for squark or slepton mass of 100 GeV.

Among the B-violating decay modes shown in the left frame of Fig. 4, $t \to c \tilde{d}_i \bar{d}_j$ has the largest branching ratio because it is a tree-level process, as shown in Fig1.(d); while other decay modes are all induced at loop-level, among which $t \to cgg$ has the largest branching ratio.

Among the L-violating decay modes shown in the right frame of Fig. 4, $t \to c \ell^-_i \ell^+_j$ also has the largest branching ratio because it is a tree-level process, as shown in Fig1.(c). The decay $t \to c \ell^-_i \ell^+_j$ also occurs at tree-level, as shown in Fig1.(a); but its branching ratio is always below $t \to c \tilde{d}_i \bar{d}_j$ for a common value of slepton and squark mass because $t \to c \tilde{d}_i \bar{d}_j$ is relatively enhanced by a color factor. Other decay modes are all induced at loop-level, among which $t \to cgg$ has the largest branching ratio.

We also calculated the channels $t \to c\gamma X (X = \gamma, Z, h)$ and found that their branching
From Fig. 4 we see that for a squark or slepton mass below 200 GeV, the decay $t \to c d_i \bar{d}_j$ can be quite sizable and could even compete with the SM decay $t \to W^+ b$. Such a large branching ratio could be readily constrained by the available data at the Tevatron. For example, the decay $t \to c d_i \bar{d}_j$ is 'exotic' to the dileptonic $t \bar{t}$ event counting because the final states of $t \bar{t}$ followed by $t \to c d_i \bar{d}_j$ and/or $\bar{t} \to \bar{c} d_i d_j$ do not have enough leptons to be included in the dileptonic event samples. In other words, only the normal decay modes of $t$ and $\bar{t}$ (i.e., $t \to W^+ b$ and $\bar{t} \to W^- \bar{b}$) can be counted into the dileptonic event samples. By comparing the CDF data [19] $\sigma[t \bar{t}]_{\exp} = 6.7 \pm 0.8(stat) \pm 0.4(syst) \pm 0.4(lumi)$ pb measured from dileptonic channels with $\sigma[t \bar{t}]_{QCD}[1 - Br(t \to c d_i \bar{d}_j)]^2$, we find that the upper bound on $B(t \to c d_i \bar{d}_j)$ given by

$$Br(t \to c d_i \bar{d}_j) \leq \begin{cases} 0.13 & (1\sigma) \\ 0.22 & (2\sigma) \end{cases}$$

(4)

FIG. 4: The branching ratios of the R-violating three-body decays of the top quark as a function of squark or slepton mass. In the left frame the product of the two $\lambda''$ couplings involved in each decay is taken as 1.2, while in the right frame the product of the two $\lambda'$ couplings involved in each decay is taken as 0.1. In the right frame the curve of $t \to c \ell_i^- \ell_j^+$ also applies to $t \to s \nu_i \ell_j^+$. Ratios are below $10^{-9}$, which are far below the detectable level of the LHC and thus are not plotted in the figures.
where we used $\sigma[t\bar{t}]_{\text{QCD}} = 7.39^{+0.57}_{-0.52}$ pb [20] and neglected the SUSY effects on the production rate [21]. Such an upper bound is plotted as the horizontal lines in Fig. 4. If we project the bound on the plane of the $\lambda''_{2ik}\lambda''_{3jk}$ versus squark mass or $\lambda''_{jk3i}\lambda''_{k3i}$ versus slepton mass, we obtain Fig. 5, where we also show the bounds from $B \to K\gamma$ and $Z$-decays [14]. We see that the Tevatron has a better sensitivity for a light squark or slepton.

![FIG. 5: The Tevatron bound shown on the plane of the relevant $\lambda''$ versus squark mass or $\lambda'$ versus slepton mass. The bounds from $B \to K\gamma$ and $Z$-decays [14] are also plotted for comparison. Above each curve is the corresponding excluded region.](image)

Since the statistical uncertainty of the top production rate will be greatly reduced at the LHC, the LHC will have better sensitivities to these exotic three-body decays. Of course, the sensitivity will be different for different decay channels. To figure out the sensitivity for each channel we need detector-dependent full Monte Carlo simulations. A preliminary fast detector simulation showed [24] that the LHC may have a high sensitivity (about $10^{-6}$) to the R-violating top decays if the decay products contain two leptons plus some jets.

IV. CONCLUSION

In the minimal supersymmetric standard model the R-parity violating interactions can induce various exotic three-body decays for the top quark, which might be accessible at the...
LHC. We collectively checked these decays, which include the tree-level processes \( t \rightarrow c\ell_i^- \ell_j^+ \) (\( \ell_i = e, \mu, \tau \)) and \( t \rightarrow cd_i d_j \) (\( d_i = d, s, b \)), as well as the loop-induced processes \( t \rightarrow cgX \) (\( X = g, \gamma, Z, h \)). We found that the weakly constrained R-violating couplings can make the decay branching ratios quite sizable, some of which already marginally reach the sensitivity of the Tevatron collider and can be explored at the LHC with better sensitivity.

Acknowledgement

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Appendix A: Expressions of loop-induced effective vertices

Here we list the expressions for the L-violating contributions to the effective vertex \( tcg \) in Fig.2. We also present the results for the effective vertices \( tc\gamma \), \( tcZ \) and \( tc\chi \), whose Feynman diagrams are similar to Fig.2. Their expressions are given by

\[
\Gamma_{tcg}^{\mu}(\tilde{l}_i^L) = \Gamma_{tcg}^{\mu}(\tilde{l}_i^L) + \Gamma_{tcg}^{\mu}(\tilde{d}_k^R), \quad \Gamma_{tcg}^{\mu}(\tilde{d}_k^R) = \Gamma_{tcg}^{\mu}(\tilde{d}_k^R),
\]

(A1)

\[
\Gamma_{tcg}^{\mu}(c\ell_i^-) = a_d \left[ C_{\alpha \beta \gamma}^{1} \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\alpha P_L - C_{\alpha}^{1}(y_h - y_\ell) \gamma^\alpha \gamma^\alpha P_L + \frac{1}{m_i^2} \gamma^\mu y_{c(\gamma^\alpha B_{\alpha}^1 P_L} \right.
\]

\[\left. - \frac{1}{m_i^2}(\gamma^\alpha y_{c(\gamma^\alpha P_L + m_\ell \gamma^\alpha \gamma_\mu P_R)B_{\alpha}^2} \right) \]

(A3)

\[
\Gamma_{tcg}^{\mu}(d_k^R) = a_d \left[-2 C_{\alpha \mu \gamma}^{4} \gamma^\alpha P_L + C_{\alpha}^{4}(p_\ell + p_e) \gamma^\alpha P_L + \frac{1}{m_i^2} \gamma^\mu y_{c(\gamma^\alpha B_{\alpha}^3 P_L} \right.
\]

\[\left. - \frac{1}{m_i^2}(\gamma^\alpha y_{c(\gamma^\alpha P_L + m_\ell \gamma^\alpha \gamma_\mu P_R)B_{\alpha}^4} \right) \]

(A4)

\[
\Gamma_{tc\gamma}^{\mu}(\tilde{l}_i^L) = ae \left\{ \frac{3W}{cW} \left[ C_{\alpha \beta \gamma}^{1} \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\alpha P_L - C_{\alpha}^{1}(y_h - y_\ell) \gamma^\alpha \gamma^\alpha P_L + \frac{1}{m_i^2} \gamma^\mu y_{c(\gamma^\alpha B_{\alpha}^1 P_L} \right. \right.
\]

\[\left. + \frac{1}{2sW}(1 - \frac{4}{3}s_W) m_i^2 \gamma^\mu y_{c(\gamma^\alpha B_{\alpha}^1 P_L} \right. \]

\[\left. + \left( \frac{s_W}{cW} - \frac{1}{2sW} \right) \left[ 2 C_{\alpha \mu \gamma}^{2} \gamma^\alpha - C_{\alpha}^{2}(p_\ell + p_e) \gamma^\alpha \right] P_L \right. \]

\[\left. - \frac{1}{2sW} \frac{1}{m_i^2} \left( (1 - \frac{4}{3}s_W) \gamma^\alpha y_{c(\gamma^\alpha P_L - \frac{4}{3}s_W m_\ell \gamma^\alpha \gamma_\mu P_R)B_{\alpha}^2} \right) \right\} \]

(A5)
\( \Gamma_{\mu}^{tcZ} (d_R^k) = a \varepsilon \left\{ \frac{1 - 2 s_W^2}{2 s_W c_W} \left( C_{a_0}^3 \gamma^\alpha \gamma^\mu \gamma^\beta - C_{a_0}^4 (y_\ell - y_{\bar{\ell}}) \gamma^\mu \gamma^\alpha \right) P_L \\
+ \frac{3 s_W}{3 c_W} \left[ -2 C_{a_0}^4 \gamma^\alpha + C_{a_0}^4 (p_t + p_c) \gamma^\mu \gamma^\alpha \right] P_L + \frac{1}{2 s_W c_W} \left( 1 - \frac{4 s_W^2}{3} \right) \frac{1}{m_t^2} \gamma^\mu y_\ell \gamma^\alpha B_{a_0}^3 P_L \\
- \frac{1}{2 s_W c_W m_t^2} \left[ \left( 1 - \frac{4 s_W^2}{3} \right) y_\ell \gamma^\alpha P_L - \frac{4 s_W^2 m_t \gamma^\alpha \gamma_{\mu} \gamma_{\mu} P_R \right] B_{a_0}^1 \right) \right\} \) \hspace{1cm} (A6)

\( \Gamma_{\mu}^{tc\gamma_1} (l_L^t) = a \varepsilon \left\{ - \frac{1}{3} \left[ C_{a_0}^3 \gamma^\alpha \gamma^\mu \gamma^\beta - C_{a_0}^4 (y_\ell - y_{\bar{\ell}}) \gamma^\mu \gamma^\alpha \right] P_L - \frac{1}{3} \left[ -2 C_{a_0}^4 \gamma^\alpha + C_{a_0}^4 (p_t + p_c) \gamma^\mu \gamma^\alpha \right] P_L \\
+ \frac{2}{3 m_t^2} \gamma^\mu y_\ell \gamma^\alpha B_{a_0}^1 P_L - \frac{1}{3 m_t^2} \left[ \gamma^\alpha y_\ell \gamma^\mu P_L + m_t \gamma^\alpha \gamma_{\mu} P_R \right] B_{a_0}^2 \right\} \) \hspace{1cm} (A7)

\( \Gamma_{\mu}^{tc\gamma_2} (d_R^k) = a \varepsilon \left\{ C_{a_0}^3 \gamma^\alpha \gamma^\mu \gamma^\beta - C_{a_0}^4 (y_\ell - y_{\bar{\ell}}) \gamma^\mu \gamma^\alpha \right] P_L - \frac{1}{3} \left[ -2 C_{a_0}^4 \gamma^\alpha + C_{a_0}^4 (p_t + p_c) \gamma^\mu \gamma^\alpha \right] P_L \\
+ \frac{2}{3 m_t^2} \gamma^\mu y_\ell \gamma^\alpha B_{a_0}^1 P_L - \frac{1}{3 m_t^2} \left[ \gamma^\alpha y_\ell \gamma^\mu P_L + m_t \gamma^\alpha \gamma_{\mu} P_R \right] B_{a_0}^2 \right\} \) \hspace{1cm} (A8)

\( \Gamma_{\mu}^{tc\beta} (l_L^t) = a \varepsilon \left\{ - m_d \sin(\alpha + \beta) C_{a_0}^3 \gamma^\alpha P_L - \frac{1}{2 s_W c_W} m_Z \sin(\alpha + \beta) \gamma^\alpha P_L \\
- Y_{l_L^t} \frac{1}{m_t^2} \left[ \gamma^\alpha y_\ell \gamma^\mu P_R + m_t \gamma^\alpha P_L \right] B_{a_0}^2 \right\} \) \hspace{1cm} (A9)

\( \Gamma_{\mu}^{tc\gamma_3} (d_R^k) = a \varepsilon \left\{ - m_t \sin(\alpha + \beta) C_{a_0}^3 \gamma^\alpha P_L + \frac{s_W}{3 c_W} m_Z \sin(\alpha + \beta) \gamma^\alpha P_L \\
- Y_{d_R^k} \frac{1}{m_t^2} \left[ \gamma^\alpha y_\ell \gamma^\mu P_R + m_t \gamma^\alpha P_L \right] B_{a_0}^2 \right\} \) \hspace{1cm} (A10)

with \( a = i \lambda_{l_L^t} \lambda_{l_L^t} \gamma_5 / (16 \pi^2) \), \( s_W = \sin \theta_W \), \( c_W = \cos \theta_W \), and \( p_t \) and \( p_c \) denoting respectively the momenta of the top and charm quark, and the Yukawa couplings given by

\[ Y_{l_L^t} = \frac{m_t \sin \alpha}{2 m_W \sin \theta_W \cos \beta}, \quad Y_{d_R^k} = \frac{m_t \sin \alpha}{2 m_W \sin \theta_W \cos \beta}, \quad Y_{l_L^t} = \frac{m_c \cos \alpha}{2 m_W \sin \theta_W \sin \beta} \] \hspace{1cm} (A11)

For the loop functions \( B \) and \( C \) in Eqs.(A1-A8), we adopt the definition in [22] and use LoopTools [23] in the calculations. The loop functions’ dependence is given by

\[ C^1 = C(-p_t, p_c, m_{d_R^k}, m_{d_R^k}, m_{d_R^k}), \quad C^2 = C(-p_t, p_t - p_c, m_{d_R^k}, m_{d_R^k}, m_{d_R^k}), \] \hspace{1cm} (A12)

\[ C^3 = C(-p_t, p_c, m_{l_L^t}, m_{d_R^k}, m_{d_R^k}), \quad C^4 = C(-p_t, p_t - p_c, m_{d_R^k}, m_{d_R^k}, m_{d_R^k}), \] \hspace{1cm} (A13)

\[ B^1 = B(-p_t, m_{d_R^k}, m_{d_R^k}), \quad B^2 = B(-p_c, m_{d_R^k}, m_{d_R^k}), \] \hspace{1cm} (A14)

\[ B^3 = B(-p_t, m_{l_L^t}, m_{d_R^k}), \quad B^4 = B(-p_c, m_{l_L^t}, m_{d_R^k}). \] \hspace{1cm} (A15)


