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Light Dark Matter from the $U(1)_X$ Sector in the NMSSM with Gauge Mediation

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Abstract: Cosmic ray anomalies observed by PAMELA and Fermi-LAT experiments may be interpreted by heavy (TeV-scale) dark matter annihilation enhanced by Sommerfeld effects mediated by a very light (sub-GeV) $U(1)_X$ gauge boson, while the recent direct searches from CoGeNT and DAMA/LIBRA experiments may indicate a rather light ($\sim$ 7 GeV) dark matter with weak interaction. Motivated by these apparently different scales, we consider a gauge mediated next-to-the minimal supersymmetric standard model (NMSSM) entended with a light $U(1)_X$ sector plus a heavy sector $(\bar{H}_h, H_h)$, which can provide both a light ($\sim$ 7 GeV) and a heavy (TeV-scale) dark matter without introducing any ad hoc new scale. Through the Yukawa coupling between H_h and the messager fields, the $U(1)_X$ gauge symmetry is broken around the GeV scale radiatively and a large negative m_S^2 is generated for the NMSSM singlet S. Furthermore, the small kinetic mixing parameter between $U(1)_X$ and $U(1)_Y$ is predicted to be $\theta \sim 10^{-5} - 10^{-6}$ after integrating out the messengers. Such a light dark matter, which can have a normal relic density from the late decay of the right-handed sneutrino (assumed to be the ordinary next-to-the lightest supersymmetric particle and thermally produced in the early Universe), can serve a good candidate to explain the recent CoGeNT and DAMA/LIBRA results.

Contents

1. Introduction

The recent indirect dark matter detection experiments like PAMELA [1] and Fermi-LAT [2] found cosmic ray anomalies, which can be interpreted by dark matter annihilation or decay (although some astrophysical explanations like pulsars are also possible). This inspires the construction of a class of models with a light dark $U(1)_X$ sector [3], which gives a sub-GeV dark gauge boson. Such a sub-GeV gauge boson plays a key role in the dark matter explanation of the cosmic ray anomalies: for the annihilating dark matter it can induce large Sommerfeld enhancement and kinetically forbid the hadronic products from the annihilation, while for the decaying dark matter [4] it can suppress the hadronic activity [5]. At the same time, some dark matter direct detection experiments such as DAMA/LIBRA [6], CDMS II [7] and CoGeNT [8] also reported some plausible evidence of dark matter, which, including the null result from XENON10 [9], may be accommodated by a quite light dark matter at GeV scale ($\sim$ 7 GeV) with a dark matter nucleon scattering cross section $\sigma_p \sim 10^{-40}$ cm^2 [10]. This has inspired some recent studies on the light dark matter [11, 12].

With Sommerfeld enhancement, it seems to us that the dark matter explanation for all these experiments must involve three very different scales: the TeV-scale heavy dark matter (HDM), the GeV-scale light dark matter (LDM), and the sub-GeV $U_X(1)$ dark sector. It is then quite challenging to embrace all these aspects in one framework. Firstly, it is not a trivial problem to accommodate such a light $U(1)_X$ gauge boson at low energy without introducing a new scale by hand. As is well known, supersymmetry (SUSY) helps to stabilize a scale and, moreover, its breaking usually generates a new scale which is encoded in the soft SUSY breaking terms. Thus the crucial task is to obtain a proper GeV-scale soft Lagrangian for the Higgs fields in the $U(1)_X$ sector. As proposed in [3] and then followed in [13, 14, 15, 16], SUSY breaking (maybe exhibited as soft terms) in some hidden sector may be gauge mediated to the $U(1)_X$ sector to generate the GeV-scale. Secondly, although it is not difficult to construct a GeV-scale $U(1)_X$ sector while allows for a sub-GeV gauge boson through introducing a very weakly charged Higgs field (say $Q_H g_X \sim 0.03$), the $U(1)_X$ sector with such a light gauge boson will usually also predict some other Higgs bosons as light as the gauge boson and the LDM annihilates to these bosons very effectively, leading to a very small relic density after freezing out (say $\Omega_{LDM}h^2 \sim 10^{-4}$). Some studies [12] showed that even with such a small relic density the LDM may still generate scattering signals at the dark matter detectors if the LDM-quark coupling strength is enhanced enough. Nevertheless, it would be more natural if the LDM density is at a normal level (~ 0.1). Thus, the LDM may be understood to be mainly produced from the late decay of the ordinary next-to-the lightest sparticle (ONLSP) in the visible sector. This may be a reasonable conjecture since in the presence of some new light $R-$odd state in the $U(1)_X$ sector, the ONLSP may decay to this sector with proper time scale.

In this work we try to extend the gauge mediated next-to-the minimal supersymmetric standard model (NMSSM) with a light $U(1)_X$ sector plus a heavy sector $(\bar{H}_h, H_h)$, which can provide both a light ($\sim$ 7 GeV) and a heavy (TeV-scale) dark matter without introducing any ad hoc new scale. In our framework the crucial dynamics is that the HDM couples directly through Yukawa couplings with the messenger fields which carry the $U(1)_X$ charge. The $U(1)_X$ gauge symmetry can be broken around the GeV scale radiatively, and a large negative m_S^2 is generated for the NMSSM singlet S. Interestingly, the small kinetic mixing parameter between $U(1)_X$ and $U(1)_Y$ is predicted to be $\theta \sim 10^{-5} - 10^{-6}$ after integrating out the messenger fields. Such a light dark matter, which can have a normal relic density from the late decay of the right-handed sneutrino, can be a good candidate to explain the recent CoGeNT and DAMA/LIBRA data.

This work is organized as follows. In Section II we present the model. In Section III we discuss its concrete realization. Finally, discussions and conclusion are given in Section IV. In Appendix A, we explain the kinetic mixing and dark-visible interaction. In Appendix B, we present the soft terms from HDM-messenger direct couplings. And in Appendix C, we give the one-loop renormalization group equations (RGEs) of some soft terms.

2. Model Building

Our model is based on the NMSSM with gauge mediated SUSY breaking (GMSB). And it has two features: (1) The NMSSM singlet S naturally provides a TeV scale to explain the origin of the HDM mass scale; (2) Through radiative correction with $1/16\pi^2$ suppression, the GMSB provides a simple way to generate the GeV-scale for the $U(1)_X$ dark sector. Some previous studies on this line have been carried out [3, 13, 14, 15, 16]. In our study we will intensively examine the dark matter phenomenology in the NMSSM extended with the $U(1)_X$ sector and the extra TeV-scale degree of freedoms, paying special attention to the mechanism of the $U(1)_X$

breaking at GeV-scale. We find that if the conventional hidden sector messengers are slightly charged under $U(1)_X$, then the soft terms in the $U(1)_X$ dark sector can be at a proper scale. Our work will address the following problems in a coherent framework:

2.1 Generating a Large Negative Soft Mass-Square for S

As a simple extension of the MSSM, the NMSSM [17] can solve the μ problem and the little hierarchy problem [18], and thus has recently attracted much attention [19]. However, in the mechanism of the GMSB it is difficult to construct a phenomenologically acceptable NMSSM [20, 21]. The key difficulty is that the singlet S couples only to the Higgs doublets and thus the soft term m_S^2 can not be generated negative enough at the weak scale through RGE. To solve this problem, some efforts have been made, e.g., coupling S to extra $SU(3)_C$-charged particles [20] or directly to messengers [22, 23, 24]. In our framework, since we have extra states $(\bar{H}_h, H_h)$ which couple to S, we can obtain large enough m_S^2 by only coupling H_h or $\bar{H}_h$ directly to messengers (S does not couple to messengers). In fact, this is a natural choice since this coupling leads to a large ($\sim$TeV^2) splitting between the soft mass-square $m^2_{\bar{H}_h}$ and $m^2_{H_h}$ at the messenger boundary. That significantly impacts on the evolution of the soft mass-square of the dark Higgs field, leading to a negative mass-square and breaking the $U(1)_X$ in the dark sector. The dynamics of this part is described by the superpotential

$$\begin{aligned} W_1 = & \left(\lambda S H_u H_d + \frac{\kappa}{3} S^3\right) + \lambda_h S \bar{H}_h H_h \\ & + \bar{H}_h \left(\lambda_T T_1 \bar{T}_2 + \lambda_D D_1 \bar{D}_2\right) + X \left(\xi_{1,T} \bar{T}_1 T_1 + \xi_{1,D} \bar{D}_1 D_1 + (1 \to 2)\right), \quad (2.1) \end{aligned}$$

where X is the spurion Goldstino field parameterized as $X = M + \theta^2 F$, and $(T_i, D_i) = f_i$ and $(\bar{T}_i, \bar{D}_i) = \bar{f}_i$ form $(5, \bar{5})$ representation of $SU(5)-$GUT group. The matter and messenger fields have the assignments under the Z_3-symmetry of the NMSSM and the $U(1)_X$:

$$\begin{aligned} & S \to e^{i\pi/3} S, \quad H_h \to e^{i2\pi/3} H_h, \quad \bar{H}_h \to \bar{H}_h, \\ & [f_1] = -[\bar{f}_1] = Q_{f_1}, \quad [\bar{H}_h] = -[H_h] = -Q_{H_h}, \quad (2.2) \end{aligned}$$

while all other fields are neutral under the above symmetries, thus $Q_{f_1} = Q_{H_h}$.

Let us comments on the superpotential:

(1) The superpotential has a Z_2^h symmetry to keep the HDM stable (the messengers $(\bar{f}_1, f_1)$ are Z_2^h-odd). According to a recent study [25], the explanation of PAMELA through such HDM annihilation with Sommerfeld enhancement is difficult. In particular, the maximal Sommerfeld enhanced factor is about 100 for a TeV scale heavy dark matter. To explain the PAMELA and Fermi-LAT experiments, for simplicity, we assume that the dark matter density in the sub-halo is about three or four times larger than the usual value. By the way, to explain PAMELA, we had better resort to decaying HDM. To let our HDM to decay to dark gauge bosons, we need to break the Z_2^h symmetry by introducing some new mechanism [5]. We will not further discuss the HDM phenomenology in this work. Instead, we will focus on the LDM phenomenology.

(2) It is important to arrange $U(1)_X$ charge to forbid coupling like $\lambda_f \bar{H}_h \bar{f}_i f_i$, which leads to a one-loop tadpole for S in the superpotential after integrating out the messengers: $\int d^2\theta \xi S$ with $\xi \sim \lambda_f^2 F/16\pi^2$, which tends to destabilize the weak scale. But at the messenger boundary, a large negative mass-square for S is generated at two loop as

$$m_S^2 = -\frac{1}{(16\pi^2)^2}\left(3\lambda_T^2 + 2\lambda_D^2\right)\lambda_h^2 \frac{F^2}{M^2}, \tag{2.3}$$

which can be as large as several-hundred GeV, depending on the couplings. For example, for $M \simeq 10^8$ GeV at the messenger boundary and taking Yukawa couplings as $\lambda_h \sim 1$, $\lambda_T \simeq \lambda_D \sim 0.2$, we have $m_S^2 \sim -(280\ \text{GeV})^2$. In this way, it is possible to make the NMSSM in the GMSB to have successful electroweak symmetry breaking.

(3) The Yukawa coupling λ_h plays an important role. In additional to generate a large m_S^2, a large λ_h is also required for having a HDM. From Eq. (C.9), the one-loop evolution of λ_h below the messenger scale is approximated as (drop the small contribution from λ, κ and $Q_h g_X$)

$$\lambda_h(M_{susy}) \approx \left(\frac{1}{\lambda_h^3(M)} - \frac{18}{16\pi^2}\log\frac{M_{susy}}{M}\right)^{-1/3}. \tag{2.4}$$

We need $\lambda_h(M_{susy}) \sim 0.5 - 1$ (depending on the value of v_s) to have a heavy HDM. Besides, it makes the HDM to annihilate to some states in the NMSSM effectively so that to have small relic density. In this way the HDM can avoid direct detection and explain the cosmic ray anomaly by a proper shorter lifetime than the decaying HDM with the assumption $\Omega_{HDM}h^2 \simeq 0.12$.

2.2 Generating A Small Negative Soft Mass-Square for the Dark Higgs H

In our model we assume that the dark sector respects a global $SU(N)$ flavor symmetry (it can be gauged to form a non-Abelian dark sector [26, 14], but in this work we do not discuss this case). This symmetry is useful because it can protect the light dark matter candidate to be stable and allow to construct a simple dark sector without anomaly if we require the dark sector has no $U(1)_X$ singlet (we will explain why we do not prefer a singlet later). The minimal field content includes: $(\bar{H}_l, H_l)$ carrying $U(1)_X$ charge $(Q_{\bar{H}_l}, Q_{H_l})$ and forming the $(\bar{N}, N)$ representation of $SU(N)$; the dark Higgs H carrying $U(1)_X$ charge Q_{H_h} and being a flavor singlet. Then, under the symmetry $U(1)_X \times SU(N)$, the most general superpotential takes a very simple form:

$$W_{dark} = \lambda_l H \bar{H}_l H_l. \tag{2.5}$$

The $U(1)_X^3$ anomaly cancellation and the $U(1)_X$ neutral condition lead to two equations:

$$Q_{H_h} + N(Q_{H_l} + Q_{\bar{H}_l}) = 0, \quad Q_{H_h}^3 + N(Q_{H_l}^3 + Q_{\bar{H}_l}^3) = 0. \tag{2.6}$$

Note that other $U(1)_X$ -charged states in our model are vector-like, and thus do not contribute to anomaly. Especially, it has a nontrivial solution

$$Q_{H_l} = -\frac{Q_{H_h}}{2N}\left(1 - \text{sign}(Q_{H_h})\sqrt{\frac{4N^2-1}{3}}\right), \tag{2.7}$$

$$Q_{\bar{H}_l} = -\frac{Q_{H_h}}{2N}\left(1 + \text{sign}(Q_{H_h})\sqrt{\frac{4N^2-1}{3}}\right). \tag{2.8}$$

Another solution is trivial, obtained by exchanging the role of $\bar{H}_l$ and H_l. For any allowed N, Q_{H_l} and $Q_{\bar{H}_l}$ take opposite sign with Q_{H_h}. This is a required property to assure that only H gets negative soft mass-square.

¿From the requirement of a negative m_H^2 at the dark scale μ_d, Q_{H_h} is determined to take the same sign with Q_{H_h}. For pure GMSB, at the messenger boundary, due to anomaly cancellation, there is a sum rule for the soft terms: $\mathcal{S}_X \equiv \text{Tr}(Q_i m_i^2) = 0$, with the trace running over all $U(1)_X$ -charged fields. But the $U(1)_X$ -charged HDM directly couples to the messengers in the hidden sector and acquires a large boundary value through Yukawa mediation (see Eqs. (B.6) and (B.7)), which violates this sum rule. Consequently, the non-vanishing $\mathcal{S}_X$ drastically changes the renormalization of the dark Higgs soft mass-squares, driving some of them negative at μ_d. The trace term is then given by

$$\begin{aligned}\mathcal{S}_X &= \mathrm{Tr}(Q_i m_i^2) = Q_{H_h}(m_{H_h}^2 - m_{\bar{H}_h}^2) \\ &\sim Q_{H_h}\frac{\lambda_T^2}{(16\pi^2)^2}\left[16g_3^2(M) - 5\lambda_h^2(M)\right]\frac{F^2}{M^2}.\end{aligned} \tag{2.9}$$

The above estimation is based on the requirement that at the scale M, $\lambda_{T,D} \ll g_3, \lambda_h$, where $g_3(M) \simeq 0.9$. There is a substantial cancellation between the terms in the bracket and thus in the following estimation we set $16g_3^2(M) - 5\lambda_h^2 \equiv C_T g_3^2(M)$ with $C_T \sim 1$. Consequently, it generates the low energy value of m_H^2 (see Eq. (C.3)):

$$m_H^2(\mu_d) \simeq m_{gauge}^2(M) + \frac{2Q_{H_h}g_X^2}{16\pi^2}\mathcal{S}_X \times \log\frac{\mu_d}{M}, \tag{2.10}$$

where the first term is the small soft term contributed by pure $U(1)_X$ gauge mediation. Then, we can parameterize the low energy dark Higgs parameter as

$$\begin{aligned}m_H^2(\mu_d) \approx &\left[0.16\left(\frac{Q_{f_1}g_X}{0.01}\right)^2\left(\frac{Q_H g_X}{0.02}\right)^2\right. \\ &\left. -0.2\left(\frac{Q_H g_X}{0.02}\right)\left(\frac{Q_{H_h}g_X}{0.01}\right)\left(\frac{\lambda_T}{0.2}\right)^2\right]\mathrm{GeV}^2.\end{aligned} \tag{2.11}$$

Here we set $F/M = 10^5$ GeV, $C_T = 0.5$ and a typical dark scale $\mu_d \sim 10$ GeV. With a moderate arrangement for $Q_{f_1}g_X$ and λ_T, we readily get $0 > m_H^2(\mu_d) \gtrsim -1\ \mathrm{GeV}^2$. Note that we do not need cancellation between the two contributions. In practice, we only require that the second term dominates over the first term and takes small value. The soft mass-square for $(\bar{H}_l, H_l)$ can be obtained similarly, which is enhanced by the trace term because their $U(1)_X$ charge is opposite to H.

Let us comment on the above charge assignments. First, $Q_{f_1}g_X \sim 0.01$ not only determines the soft mass scale from $U(1)_X$ mediation, but also directly relates with the value of θ discussed later. Next, a small value $Q_{H_h}g_X \sim 0.01$ ont only helps to make the HDM to avoid direct detection, but also avoid the unnecessary enhancement by dark gauge boson, which will subject to the gamma ray constraint [1]. As for the small $Q_H g_X \sim 0.02$, controlling the quartic term from D-term, is necessary to generate a larger VEV of the dark Higgs, providing the several-GeVs scale for the light dark matter.

[1]In principle, we can increase $Q_h g_X \sim 0.5$ and meanwhile set a smaller value $\lambda_T \sim 0.03$. It makes the HDM still be a candidate for annihilating dark matter with Sommerfeld enhancement.

2.3 Predicting a Small Kinetic Mixing Parameter θ

Since the messengers also carry $U(1)_X$ charges, our framework naturally predicts a value for the kinetic mixing parameter θ between $U(1)_X$ and $U_Y(1)$:

$$\theta \sim \sum_I n_I \frac{g_1 g_X Q_Y^I Q_X^I}{16\pi^2} \log \frac{M_{GUT}}{M}, \tag{2.12}$$

where n_I is the number of fields I that carry hypercharge Q_Y^I and $U(1)_X$ charge Q_X^I. At first, the contribution from a complete representation of $SU(5)$ cancels exactly due to the traceless generators of $SU(5)$. For example, for $(\bar{f}_1, f_1)$ we have

$$2 \times \left(3 \times \frac{1}{3} \times Q_{f_1} + 2 \times \left(-\frac{1}{2}\right) \times Q_{f_1}\right) = 2Q_{f_1} \mathrm{Tr}(T_{24}) = 0, \tag{2.13}$$

with T_{24} a generator of $SU(5)$ that defines hypercharge. In general, the small doublet-triplet splittings between messenger fields can be obtained after the $SU(5)$ gauge symmetry breaking via the dimension-5 operators $X\bar{f}_i\Phi f_i/\Lambda$ (with operator coefficient set to be 1) where Φ is the $\mathbf{2}4$ representation Higgs field and Λ is the fundamental scale of the UV theory such as string scale $M_{string} \simeq 3.0 \times 10^{17}$ GeV, or reduced Planck scale $M_{\rm Pl} \simeq 2.4 \times 10^{18}$ GeV [27]. Note that the $SU(5)$ unification scale is about 2.4×10^{16} GeV, thus if we take $\Lambda = M_{string}$, we obtain $|\xi_{1,D} - \xi_{1,T}|/\xi_{1,D} \sim 0.1$. In addition, the θ parameter is given by

$$\theta \simeq 2Q_{f_1} \frac{g_1 g_X}{16\pi^2} \log \frac{\xi_{1,T}}{\xi_{1,D}} \sim \mathcal{O}(10^{-5}) \ . \tag{2.14}$$

Thus, we require $Q_{f_1} g_X \sim 0.05$, which is consistent with the previous parameterization. By the way, the RGE effects may also induce the doublet-triplet splitting (see Eqs. (C.12) and (C.13)), which could be very small if the corresponding Yukawa couplings are small. On the other hand, for large Yukawa couplings, proper splitting is induced even without turning back to the high dimension operators.

2.4 Light Dark Matter Candidate

Recently, dark matter direct detection experiments showed some hints on light dark matter ~ 7 GeV. It is natural to relate it with the light $U(1)_X$ dark sector [12, 28]. Although there is a small gap between the LDM and dark gauge boson mass scale, it can be explained by a small gauge coupling of the dark Higgs ($Q_H g_X \sim 0.02$), provided that the Yukawa coupling λ_l is about 0.5. We will elaborate this problem in the next Section.

In summary, we depict our dynamics structure in Fig. 1. The hidden sector plays a crucial role in our framework: it not only generates all the necessary low energy mass scales, but also explains a small θ in the dark matter phenomenology.

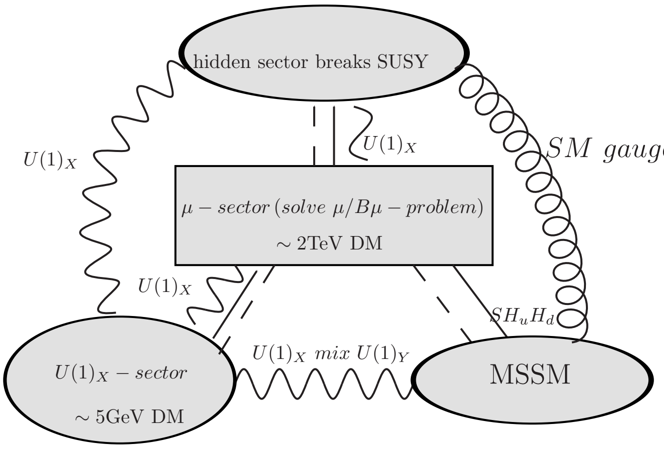

Figure 1: The schematic diagram showing our dynamics. Solid and dashed lines denote other possible interactions beyond gauge interactions.

3. Light Dark Matter Phenomenology

3.1 Vacuum and Spectra of the Theory

First we check the vacua of the visible sector and the dark sector. For the former, the desired electroweak vaccum takes a form $v_s = \langle S \rangle$, $v_u = \langle H_u^0 \rangle$ and $v_d = \langle H_d^0 \rangle$. In order to ensure the stability of the HDM and avoid the breaking of $U(1)_X$ at TeV scale, we must have $\langle \bar{H}_h \rangle = \langle H_h \rangle = 0$. We need to be cautious about this because $m^2_{H_h}(M)$ and $m^2_{\bar{H}_h}(M)$ are negative and take roughly a value as $m^2_S(M)$ (see Eqs. (B.6) and (B.7)). But a negative mass-square does not always mean a non-zero VEV in the multi-Higgs system. We can prove it by assuming a vacuum with vanishing $\langle H_h \rangle$ and $\langle \bar{H}_h \rangle$, and then check whether such a vacuum leads to a tachyonic direction. In practice, in the complex scalar mass system of $(\bar{H}_h, H_h^*)$, the mass matrix reads

$$M^2_{H_h} = \begin{pmatrix} (\lambda_h v_s)^2 + m^2_{\bar{H}_h} & (\lambda_h v_s)(\kappa\, v_s) + (\lambda_h v_s) A_{\lambda_h} \\ (\lambda_h v_s)(\kappa\, v_s) + \lambda_h v_s A_{\lambda_h} & (\lambda_h v_s)^2 + m^2_{H_h} \end{pmatrix}. \qquad (3.1)$$

Obviously, it is definitively positive provided that $\lambda_h v_s \sim 2$ TeV is much larger than other scales in the matrix. This condition can be satisfied for the following reasons. First, from the previous parameter estimation, all the soft masses typically lie much below TeV. Furthermore, to generate a large v_s we require $\kappa \sim 0.1$ ($\ll \lambda_h$). Concretely, the lightest boson has a mass-square approximated by

$$M_-^2 \approx (\lambda_h v_s)^2 + m_{\bar{H}_h}^2 - \frac{(M_{H_h}^2)_{12}^2}{m_{H_h}^2 - m_{\bar{H}_h}^2}. \tag{3.2}$$

where $(M_{H_h}^2)_{12}$ is the 12-element of $M_{H_h}^2$. This approximation is valid when $m_{H_h}^2 - m_{\bar{H}_h}^2 > |(M_{H_h}^2)_{12}|$. So, in general this is a stable dark matter in the TeV region required by PAMELA and Fermi-LAT.

We further briefly comment on the pattern of the parameter space and the symmetry breaking in the NMSSM in our scenario. First, A_λ and A_κ are induced by RGEs, suppressed by loop factor. Note that a new contribution $A_{\lambda_h} \sim -100$ GeV (see Eq. (B.5)) affects the running of A_λ and A_κ significantly:

$$16\pi^2 \frac{dA_\lambda}{dt} \approx \left(2\lambda_h^2 A_{\lambda_h} + 6h_t^2 A_t + 6g_2^2 M_2 + ...\right), \tag{3.3}$$

$$16\pi^2 \frac{dA_\kappa}{dt} \approx \left(2\lambda_h^2 A_{\lambda_h} + 12\lambda^2 A_\lambda + ...\right). \tag{3.4}$$

Since λ_h is large in our framework, this new contribution is quite sizable. Especially, A_κ will get a new contribution at order $\sim -2\frac{\left(3\lambda_T^2+2\lambda_D^2\right)\lambda_h^2}{(16\pi^2)^2}\frac{F}{M}\log\frac{\mu_d}{M} \sim \mathcal{O}(10)$ GeV. As a result, in general one can not expect a very light $R-$axion (CP-odd) a in the spectrum. But in case of small κ and λ, and $v_s \gg v$, some parameter space still allows for $m_a < 2m_b$ and consequently the $R-$axion solution to the fine-tuning problem may be accommodated [18]. Anyway, these trilinear terms are small compared with m_S^2, and thus the electroweak and Z_3 breaking in the NMSSM is dominantly driven by the negative m_S^2. Approximately, we get a $v_s \gg v$ limit through

$$v_s \simeq \frac{m_S}{\kappa} \sim \mathcal{O}(5)\,\text{TeV}, \tag{3.5}$$

which is readily achieved by a small $\kappa \sim 0.1$. Incidentally, a small κ is a safe choice to stabilize the HDM mass scale and moreover is favored by keeping the theory perturbative up to the GUT scale. In conclusion, with the effects of the Yukawa coupling between the HDM and the messengers, our scenario is capable of providing a proper solution to the NMSSM.

The $U(1)_X$ symmetry breaking and the spectrum in the dark sector can be analytically studied. The total scalar potential is $V = V_D + V_F + V_{soft}$, with each term given by [2]

$$V_D = \frac{1}{2} g_X^2 \left[\left(Q_{H_l} |H_l|^2 + Q_{\bar{H}_l} |\bar{H}_l|^2 + Q_{H_h} |H|^2 \right) + \xi_X \right]^2 , \tag{3.6}$$

$$V_F = \left| \lambda_l H_l \bar{H}_l \right|^2 + |\lambda_l|^2 |H|^2 \left(|\bar{H}_l|^2 + |H_l|^2 \right) , \tag{3.7}$$

$$V_{soft} = m_{H_l}^2 |H_l|^2 + m_{\bar{H}_l}^2 |\bar{H}_l|^2 + m_H^2 |H|^2 + \left(\lambda_l A_{\lambda_l} H \bar{H}_l H_l + h.c. \right) . \tag{3.8}$$

Among the soft terms, $m_{\bar{H}_l}^2$ and $m_{H_l}^2$ are positive while m_H^2 is negative. A_{λ_l} is purely RGE induced, roughly given by (see Eq. (C.6))

$$A_{\lambda_l} \simeq -\frac{8}{16\pi^2} (Q_{H_l} g_X)^2 m_{\widetilde{X}} \log \frac{M}{\mu_d} \sim -10^{-2}\,\text{GeV}, \tag{3.9}$$

which is much smaller than the typical scale in the dark sector and thus is not a relevant soft parameter although it controls the mixing between H_l and $\bar{H}_l^*$. H is the Higgs field which breaks $U(1)_X$ gauge symmetry. Its potential is simply a complex ϕ^4 theory, where the negative m_H^2 and quartic term from $D-$term stabilizes the potential at the minimum

$$\langle H \rangle = v_H = \frac{|m_H|}{Q_H g_X} \sim \mathcal{O}(10)\,\text{GeV}. \tag{3.10}$$

The dark spectrum can be at the required several-GeV scale simply by setting $\lambda_l \sim 0.5$. The dark gauge boson mass is given by $m_X = \sqrt{2} Q_H g_X v_H = \sqrt{2} |m_H| \simeq 0.2$ GeV, depending only on the negative Higgs parameter. To calculate the dark spectrum and the interactions in the dark sector, we take a unitary gauge to eliminate the Goldstone boson from the spectrum and write

$$H = v_H + \frac{H_R}{\sqrt{2}}. \tag{3.11}$$

The CP-even state H_R does not mix with other states and gets its mass from the quartic term ($D-$term). Since the $D-$term is determined by gauge coupling, at tree-level H_R is exactly as light as the dark gauge boson. The LDM can annihilate into such light bosons too effectively and thus the freeze-out relic density is too low, which will be discussed in the next section.

Now we study the states from the superfields $(\bar{H}_l, H_l)$. The mass-square matrix of the complex scalars in the basis of $(H_l^*, \bar{H}_l)$ is given by

$$M_l^2 = \begin{pmatrix} \lambda_l^2 v_H^2 + m_{H_l}^2 & \lambda_l A_{\lambda_l} v_H \\ \lambda_l A_{\lambda_l} v_H & \lambda_l^2 v_H^2 + m_{\bar{H}_l}^2 \end{pmatrix} . \tag{3.12}$$

[2]The study in [15] observes that the effective FI-term $\xi_I \propto \theta$ generated by the mixing $D-$term between $U(1)_Y$ and $U(1)_X$ is able to generate the proper $U(1)_X$ breaking, which is ignored in our study because θ is small.

Here we do not consider CP-violation and thus all parameters are taken as real. The soft trilinear term is only a RGE effect and is a small perturbation to the diagonal elements. So the two mass eigenstates are approximately same as the interaction states $(H_l^*, \bar{H}_l)$, with the eigenvalues given by the two diagonal elements in M_l^2. Because of the positivity of $m_{H_l}^2$ and $m_{\bar{H}_l}^2$, these bosons are heavier than the Dirac fermion formed by the two Weyl fermions $\chi = (\widetilde{H}_l, \widetilde{\bar{H}}_l)$, whose mass is given by $M_L \equiv \lambda_l v_H$. Such a $U(1)_X$-charged Dirac fermion χ serves as the LDM candidate.

Note that in our framework we do not choose a singlet type dark sector as in [15], where the dark sector superpotential is $NH'H$, with N being a singlet and $Q_{H'} = -Q_{H_h}$, and no $SU(N)$ flavor symmetry is introduced. Since in that case N is a singlet with $m_N^2 < 0$ at μ_d obtained from renormalization only, the LDM is always a singlet-like scalar and its couplings with quarks are suppressed by an extra mixing factor $1/\delta_A^2$, with $1/\delta_A$ measuring the fraction of the charged component H' in the LDM, given by

$$\delta_A \approx \frac{m_{\bar{H}}^2 - m_N^2}{|\lambda_l A_{\lambda_l} v_H|} \sim \frac{2\sqrt{2}}{\lambda_l g_X} \sim \mathcal{O}(10^2). \tag{3.13}$$

In this estimation we assumed an ideal case that the pure $U(1)_X$ mediation contribution to m_H^2 and $m_{\bar{H}}^2$ equals to the renormalization contribution from $\mathcal{S}_X$. So in that scenario, besides the suppression from $\theta^2 \sim 10^{-10}$, the LDM-nucleon scattering will be further suppressed by a factor $1/\delta_A^4 \sim 10^{-8}$, rendering the cross section unacceptably small.

Finally, we comment on $\widetilde{H}$ (fermionic component of H) and dark gaugino $\widetilde{X}$. They have a Dirac mass $m_{HX} = m_X \simeq 0.2$ GeV. And $\widetilde{X}$ also has a heavier Majorana mass term $m_{\widetilde{X}} \sim 0.5$ GeV for the choice $Q_{f_1} g_X \simeq 0.01$. This will lead to a seesaw-like spectrum, *i.e.*, the lighter one is very light, even as light as tens of MeV. Provided that the SUSY breaking scale is high enough, saying $\sqrt{F} \sim 10^9$ GeV, this particle will be the LSP (otherwise, we have to make sure that after decoupling, it decays away before the BBN).

3.2 Light Dark Matter Relic Density

In this paper we mainly discuss the LDM phenomenology and try to explain the CoGeNT and DAMA/LIBRA results together with other null results from XENON100, XENON10 and CDMS(Si). We use the latest data analysis in [10], which showed that the combination of DAMA/LIBRA and CoGeNT data can be well accommodated by a LDM with a mass of ~ 7 GeV and an elastic scattering cross section with the nucleon of $\sim 2 \times 10^{-40}$ cm^2. Moreover, it showed that such a LDM is not excluded by other null results.

In our framework we have such a LDM from the dark sector, the Dirac fermion χ. However, a proper relic density for this LDM is hard to obtain from the standard freeze-out thermal production. In fact, there are two annihilation channels for this LDM: one is directly to the dark gauge boson with a rate $\propto 4\pi(Q_{H_l} g_X)^4/m_L^2$ and the other is to H_R with a rate $\propto 4\pi\lambda_l^4/m_L^2$. Clearly, without the suppression of any large mass scale (e.g., a weak scale heavy field in the propagator), the only way to keep the LDM as a weakly interacting massive particle (WIMP) with a typical weak reaction rate $\sigma_0 \sim 10^{-8}$ GeV^{-2} is to set $Q_{H_l} g_X, \lambda_l \lesssim 0.03$. But such a smaller λ_l implies v_H must take several hundred GeVs to keep the mass of χ is about 7 GeV. Anyway, in principle it is a viable solution, for example, by taking

$$Q_H g_X \simeq 0.001, \quad \lambda_l \simeq 0.03, \quad \theta \simeq 10^{-4}, \tag{3.14}$$

and keep m_X as light as 0.2 GeV, *i.e.*, $m_H^2(\mu_d) \simeq -0.04$ GeV^2. But from Eq. (2.11) we have to choose $Q_{f_1} g_X \simeq Q_{H_h} g_X \simeq 0.2$. Although this solution has a virtue that it allows the HDM to be the Sommerfeld type instead of decaying HDM (since HDM couples to dark gauge boson with a large strength), it is at the price of a surprisingly large $U(1)_X$ charge hierarchy between the different fields, *e.g.*, $Q_{H_l} : Q_{H_h} = 1 : 200$. So we propose that the LDM abundance is produced by the late-decay of the ordinary next-to-the lightest supersymmetric particle (ONLSP) in the NMSSM (the collider phenomenology of such ONLSP decay to dark states is studied in [14]). In this solution, λ_l takes a large value so that the LDM annihilates to H_R very fast and eventually leaves a small abundance after decoupling. However, in the presence of a light $U(1)_X$ sector, the ONLSP will dominantly decay to the $U(1)_X$-charged dark states. If this decay happens after the decoupling of the LDM (typically $\sim 10^{-5}s$), then the number density of the ONLSP is transferred to the LDM. Of course, a rather large number density is needed because the relic energy density of the LDM is proportional to its mass. But if the ONLSP has a very weak annihilation, its relic number density can be quite large. In the following we discuss this issue quantitatively.

First we consider the lightest neutralino N_1 as the ONLSP. If N_1 dominantly decays to dark Higgsino and dark Higgs, its lifetime is estimated to be

$$\tau_{N_1\to h+\widetilde{h}} \sim \left(Q_{H_l}^2\alpha_X f_{\widetilde{B}}^2\theta^2 M_{N_1}\right)^{-1}$$
$$\simeq 7\times 10^{-14}\times\left(\frac{10^{-5}}{\theta}\right)^2\left(\frac{10^{-3}}{Q_{H_l}^2\alpha_X}\right)\left(\frac{1}{f_{\widetilde{B}}}\right)^2\left(\frac{100\,\mathrm{GeV}}{M_{N_1}}\right)s, \quad (3.15)$$

where $f_{\widetilde{B}}$ is the fraction of bino in N_1. In order for this decay to be late enough, the bino component should be highly suppressed $\sim 10^{-4}$.

Then we assume the right-handed snuetrino (sRHN) as the ONLSP. Such a sRHN is present in the NMSSM extended with a right-handed neutrino, which was used to explain the light neutrino masses by seesaw mechanism [29, 30]. We consider a simple model with only one flavor RHN (denoted as N) and lepton doublet introduced. Its relevant superpotential is given by

$$W_N = Y^N L H_u N + \frac{M_N}{2}N^2 + \frac{\lambda_{SN}}{2}SN^2 + \mu H_u H_d, \quad (3.16)$$

where $\mu\equiv\lambda v_s$. Depending on the Z_3-charge assignment, M_N or λ_{SN} can be turned off. In the following we focus on the case with $\lambda_{SN}=0$. Further, in GMSB the soft terms involving the SM singlet N can be dropped safely because they are all generated by RGE effects from the coupling to L and H_u (such renormalization effect is suppressed by $Y^N\sim 10^{-5}$ in the low-scale seesaw). The LR-mixing is naturally suppressed. In the CP-eigenstate basis of sleptons $(\widetilde{\nu}_+^*, \widetilde{N}_+^*, \widetilde{\nu}_-^*, \widetilde{N}_-^*)$, the mass matrix is

$$M_{\widetilde{\ell}}^2 \approx \begin{pmatrix} m_{\widetilde{\ell}}^2 + D^2 & F^2 + m_D M_N & 0 & 0 \\ & m_{\widetilde{N}}^2 + M_N^2 + B_N M_N & 0 & 0 \\ & & m_{\widetilde{\ell}}^2 + D^2 & F^2 - m_D M_N \\ & & & m_{\widetilde{N}}^2 + M_N^2 - B_N M_N \end{pmatrix}, \quad (3.17)$$

where $D^2 = 0.5 m_Z^2\cos(2\beta)$ and the mixing parameters $F^2\approx -\mu m_D\cot\beta$ [31]. The lightest state $\widetilde{\nu}_1$ with mass-square $m_{\widetilde{\nu}_1}$ is dominated by $\widetilde{N}_-^*$, provided that

$$m_{\widetilde{\nu}_1}^2 \approx m_{\widetilde{N}}^2 + M_N^2 - B_N M_N \approx M_N^2 < m_{\widetilde{\ell}}^2 + D^2. \quad (3.18)$$

We have used the fact that the splitting is small, so the mass eigenvalues are nearly the four diagonal elements. Concretely, the component of $\widetilde{\nu}_-^*$ is given by $\widetilde{\nu}_1\supset \mathcal{C}_1^-\widetilde{\nu}_-^*$ with

$$\mathcal{C}_1^- \approx \frac{F^2 - m_D M_N}{\delta m_{12}^2} \approx \frac{F^2 - m_D M_N}{m_{\widetilde{\ell}}^2 - M_N^2}, \quad (3.19)$$

where δm_{12}^2 is the mass-square splitting between the two mass eigenstates of 3-4 block in the matrix Eq. (3.17). Depending on the mass splitting and M_N, the fraction covers over a wide region:

$$|\mathcal{C}_1^-| \simeq \frac{\sqrt{m_\nu M_N^3}}{\delta m_{12}^2} \sim 10^{-8} - 10^{-2}, \tag{3.20}$$

where we used the seesaw formula for the light neutrino mass scale $m_\nu = m_D^2/M_N \sim 0.1$ eV.

As the ONLSP, the $\widetilde{\nu}_1$ has two decay channels to dark sector through its left-handed slepton component. One is the interesting three-body decay via $\widetilde{\nu}_1 \to \nu_L + H_l + \widetilde{H}_l$ mediated by bino, as shown in Fig. 2, and the decay lifetime is [14]

$$\begin{aligned}\tau_{\widetilde{\nu}_1} \sim& \left(Q_{H_l}^2 \alpha_X f_{\widetilde{B}}^2 (\mathcal{C}_1^-)^2 \theta^2 \frac{m_{\widetilde{\nu}_1}}{16\pi^2 P(m_{\widetilde{\nu}_1}/M_1)}\right)^{-1} \\ \simeq& 2.6\times 10^{-3} s \times \left(\frac{10^{-4}}{f_{\widetilde{B}}\mathcal{C}_1^-}\right)^2 \left(\frac{10^{-5}}{\theta}\right)^2 \frac{10^{-3}}{Q_{H_l}^2\alpha_X} \frac{300\,\mathrm{GeV}}{m_{\widetilde{\nu}_1}} \frac{1}{P(m_{\widetilde{\nu}_1}/M_1)}.\end{aligned} \tag{3.21}$$

The other channel is $\widetilde{\nu}_1 \to \nu_L + \widetilde{X}$ (also see Fig. 2), but is suppressed by an additional helicity factor $(m_{\widetilde{X}}/M_1)^2$ and typically several times smaller than the three-body decay [14]. Moreover, it can also the decay into Goldstino $\widetilde{\nu}_1 \to \nu_L + \widetilde{G}$, as shown in Fig. 2, which is suppressed by the SUSY-breaking scale $\sqrt{F} \gtrsim 10^3$ TeV. So, the $\widetilde{\nu}_1$ ONLSP mainly decays to the $U(1)_X$ charged dark states before BBN era ($\gtrsim 1s$) and thus can provide a proper LDM density.

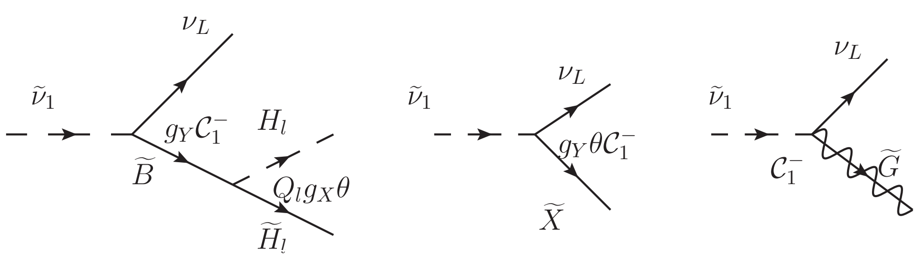

Figure 2: The $\widetilde{\nu}_1$ decays to dark states and gravitino.

To end up this section, we point out one merit of the LDM from late decay. In Ref. [32] it was shown that if the LDM reaches its relic density via annihilating to SM fermions, then the required LDM-nucleon scattering cross section generally implies antiproton excess, leading to some tension. But obviously our LDM scenario evades this constraint.

3.3 Explanation of CoGeNT and DAMA/LIBRA Results

The study in [10] suggests a ~ 7 GeV light dark matter with an elastic scattering cross section with the nucleon of $\sim 2\times 10^{-40}$ cm^2. In the following we study the LDM interaction with the nucleon.

Using the method described in [33], we derive the effective interaction between the LDM and the nucleus. The microscopic interaction is presented in Appendix. A. Due to the kinetic mixing, the LDM interacts with quarks, mediated by the dark gauge boson. It is the basis of the effective theory describing the LDM-nucleon interaction. This effective theory is obtained by calculating the quark and gluon operators in a nucleon state, such as $\langle n|\bar{f}\gamma_\mu f|n\rangle$. Then we obtain an interaction:

$$\begin{aligned}\mathcal{L}^{n,p}_{vec} &= J^\mu_X\left(X_\mu + \theta s_W Z_\mu\right) + f_n X_\mu \overline{n}\gamma^\mu n + f_p X_\mu \overline{p}\gamma^\mu p,\\ J^\mu_X &= \frac{1}{2}(Q_{H_l}+Q_{\bar{H}_l})g_X\bar{\chi}\gamma^\mu\chi + \frac{1}{2}(Q_{H_l}-Q_{\bar{H}_l})g_X\bar{\chi}\gamma^\mu\gamma_5\chi\\ &\quad + iQ_l g_X\left(\partial^\mu H_l H_l^\dagger - \partial^\mu H_l^\dagger H_l\right) + \cdots ,\end{aligned} \tag{3.22}$$

where the dots denote the irrelevant contributions like the gauginos. From Eq. (2.7) we get $Q_{H_l}+Q_{\bar{H}_l} = Q_{H_h}$. And the term proportional to $|Q_{H_l}-Q_{\bar{H}_l}| = \sqrt{\frac{4-N}{3N}}|Q_{H_h}| < |Q_{H_l}+Q_{\bar{H}_l}|$ induces spin-dependent scattering. But it is suppressed by the smaller charge $|Q_{H_l}-Q_{\bar{H}_l}|$. Moreover, for the nucleus with large atomic number $A > 20$, it is usually dominated by spin-independent scattering [33]. Thus in the following discussion we only keep the contribution from spin-independent scattering. Note that the LDM-quark interaction is mediated by the dark gauge boson. Consequently, due to the conservation of the vector current, the sea quarks and the gluons will not contribute to the current operator $\bar{f}\gamma_\mu f$. As a result, the derivation of the effective theory is not only simplified, but also free of uncertainty from considerations like spin or strangeness content of the nucleon. This implies that the effective $U(1)_X$ charge of the nucleon, $f_{p,n}$, only receives contribution from its constituent quark [33]. So we have

$$f_n = b_u + 2b_d, \quad f_p = 2b_u + b_d, \tag{3.23}$$

where $b_{u,d} = \theta g_Y \cos^2\theta_W Q_{u,d}$ with $Q_{u,d}$ being the QED-charge of u, d quarks. Thus at the leading order only the QED-charged proton carries a tiny $U(1)_X$ charge. The vector interaction only mediates the spin-independent interaction between dark matter and the nucleon and thus the LDM-nucleon scattering cross section can be added coherently to give the total LDM-nucleus cross section. That means that given the LDM-nucleon cross section σ_p, the LDM-nucleus cross section is proportional to $(Zf_p + (A-Z)f_n)^2 \sigma_p$ with Z and A being respectively the proton and atomic numbers of the nucleus.

In practice, the four-fermion effective interaction is enough for the calculation of the LDM-nucleus cross section since, for each scattering by exchanging a dark gauge boson, the typical transferred momentum is $(p_1 - p_2)^2 = |q|^2 = 2\mu_N^2 v^2(1-\cos\theta) \sim \mathcal{O}(10^{-4})\text{GeV}^2 \ll m_X^2 \simeq 4 \times 10^{-2}$ GeV^2. The reduced mass is $\mu_N = m_N m_L/(m_L + m_N) \simeq m_L$ for large nucleus like Ge with mass $m_N \approx 73$ GeV. Thus the X boson can be integrated out.

From Eq. (3.22) we calculate the scalar spin-independent scattering cross section between the LDM and the proton. It is given by

$$\sigma_p \approx \frac{N}{4}\frac{\mu_p^2}{\pi}\frac{(Q_H g_X)^2}{m_X^4} f_p^2, \tag{3.24}$$

with N being the internal index from $SU(N)$. This result is valid only in the non-relativistic limit (zero momentum transfer). Note that in most previous studies the results are usually displayed on the plane of DM mass versus the DM-nucleon scattering cross section by setting $f_p = f_n$. But in our model, $f_n \approx 0$, and thus σ_p should be re-scaled as $\sigma_p Z^2/A^2\varpi$ when compared with data, where ϖ is the fraction of LDM in the total DM. Then we have

$$\begin{aligned}\sigma_p &\to \left(\frac{Z^2}{A^2\varpi}\right)\frac{N}{4}\frac{m_p^2}{m_X^4}\frac{g_1^2 Q_{H_h}^2 g_X^2}{\pi}\theta^2\cos\theta_W^4 \\ &\sim \left(\frac{Z^2}{A^2\varpi}\right) \times 1.2 \times 10^{-40} \times N \left(\frac{0.2\,\text{GeV}}{m_X}\right)^4 \left(\frac{Q_H g_X}{0.02}\right)^2 \left(\frac{\theta}{2\times 10^{-5}}\right)^2 \text{cm}^2. \end{aligned} \tag{3.25}$$

The cross section is independent of the LDM mass. Different experiments have different values for the ratio $(Z/A)^2$. For a LDM with a mass of ~ 7 GeV, this cross section is just at the right order, according to the analysis in [10].

4. Conclusion

Both the cosmic ray anomalies observed by PAMELA and Fermi-LAT experiments and the possible events from direct detections like CoGeNT and CDMS II experiments may indicate the existence of dark matters. But the former points to a heavy dark matter at TeV sacle, while the later favors a light dark matter with a mass of several GeV. Meanwhile, the Sommerfeld enhancement may imply a dark

$U(1)_X$ gauge boson with a sub-GeV mass. In light of these apparently different mass scales, we in this work constructed a simple and coherent framework with GMSB, based on the NMSSM extended with a light $U(1)_X$ sector and a heavy dark matter sector. By coupling the heavy dark matter directly to the $U(1)_X$ -charged messengers in the hidden sector, our framework has the following intriguing features:

(1) The kinetic mixing $\theta \sim 10^{-5}$ is obtained after integrating out the messengers with small doublet-triplet splitting.

(2) A large negative mass-square m_S^2 for the NMSSM singlet S is generated at the messenger scale M.

(3) The dark $U(1)_X$ is spontaneously broken at the GeV-scale, while the dark gauge boson can have a sub-GeV mass by assuming a weakly $U(1)_X$ -charged Higgs field.

(4) A GeV-scale light dark matter with the required interaction strength with quarks is provided. And its normal relic density can be generated by the ONLSP late decay.

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A. Kinetic Mixing and Dark-Visible Interaction

The messengers in the hidden sector are charged under $U(1)_Y \times U(1)_X$. After the $SU(5)-$ gauge group is broken, these fields generate the kinetic mixing between the two groups. The effective theory (for the bosonic gauge part only, with fermionic part obtained similarly) below the messenger threshold is given by

$$\mathcal{L}_{gauge} = -\frac{1}{4} F_Y^{\mu\nu} F_{Y\mu\nu} - \frac{1}{4} F_X^{\mu\nu} F_{X\mu\nu} + \frac{\theta}{2} F_Y^{\mu\nu} F_{X\mu\nu}. \tag{A.1}$$

Setting the initial value $\theta = 0$ at M_{GUT}, then we get the value at the messenger scale M through one-loop RGE [34]

$$16\pi^2 \frac{d\theta}{dt} \approx 2 \sum_I g_X g_1 Q_Y^I Q_X^I, \tag{A.2}$$

where I runs over all the superfields with $U(1)_Y \times U(1)_X$ charge. After electroweak symmetry breaking, the three gauge bosons Z'_μ, B'_μ and X'_μ mix with each other through kinematic terms and mass terms. Using the convention in [14], after eliminating the mixing and working in the mass eigenstate basis (Z_μ, A_μ, X_μ), the interactions between the gauge boson and the current at the leading order of θ are described by

$$\begin{aligned}\mathcal{L}_{coupling} \supset\ & \theta X_\mu \left(\cos\theta_W J^\mu_{em} + \mathcal{O}(m_X^2/m_Z^2) J^\mu_Z\right) + \theta Z_\mu \left(-\sin\theta_W J^\mu_X + \mathcal{O}(m_X^2/m_Z^2) J^\mu_W\right) \\ & + \theta \left(\widetilde{B}\widetilde{J}_X + \mathcal{O}(m_{\widetilde{X}}/M_1)\widetilde{X}\widetilde{J}_B\right),\end{aligned} \tag{A.3}$$

where the bosonic gauge current $J^\mu_{em,W,Z}$ and J^μ_X are defined as usual, while its fermonic counterpart, *i.e.*, the supercurrents, are defined as $\widetilde{J}_X = g_X \sum_i Q_{X,i} \widetilde{d}_i^\dagger d_i$, $\widetilde{J}_B = g_Y \sum_i Q_{Y,i} \widetilde{v}_i^\dagger v_i$ with d_i/v_i denoting any dark/visible fermions. The $X_\mu J^\mu_{em}$ accounts for the cosmic ray anomaly after HDM decays/annihilates to the dark sector. And $\widetilde{B}\widetilde{J}_X$ provides the $U(1)_Y$ gaugino interaction with the dark states.

B. Soft Terms from HDM-Messenger Direct Couplings

If some fields feel the SUSY-breaking via direct couplings to the messengers, they will give new contribution to soft terms controlled by Yukawa couplings. The soft terms can be extracted by using wave-function renormalization method [22]. According to this method, we need to know the discontinuity of anomalous dimensions and the beta function across the messenger threshold [24]. Above the messenger scale we have

$$\begin{aligned}
\gamma_{H_h} =&\frac{1}{16\pi^2}\left(-2\lambda_h^2+4g_X^2Q_{H_h}^2\right),\\
\gamma_{\bar{H}_h} =&\frac{1}{16\pi^2}\left(-2\lambda_h^2-6\lambda_T^2-4\lambda_D^2+4g_X^2Q_{H_h}^2\right),\\
\gamma_S =&\frac{1}{16\pi^2}\left(-4\kappa^2-2\lambda_h^2-2\lambda^2\right),
\end{aligned}\tag{B.1}$$

$$\begin{aligned}
\beta_{\lambda_h} =&\frac{2\lambda_h^2}{16\pi^2}\left(2\kappa^2+3\lambda_h^2+\lambda^2+3\lambda_T^2+2\lambda_D^2-4g_X^2Q_{H_h}^2\right),\\
\beta_{\lambda_T} =&\frac{2\lambda_T^2}{16\pi^2}\left(5\lambda_T^2+\lambda_h^2+2\lambda_D^2-2g_X^2(Q_{f_1}^2+Q_{H_h}^2)-\frac{4}{9}g_1^2-\frac{16}{3}g_3^2\right),\\
\beta_{\lambda_D} =&\frac{2\lambda_D^2}{16\pi^2}\left(3\lambda_T^2+\lambda_h^2+4\lambda_D^2-2g_X^2(Q_{f_1}^2+Q_{H_h}^2)-g_1^2-3g_2^2\right).
\end{aligned}\tag{B.2}$$

In the calculation, X is taken as a non-propagating background. Across the messenger threshold, the various discontinuity is given by

$$\begin{aligned}
\Delta\gamma_{H_h} =&0,\quad \Delta\gamma_{\bar{H}_h}=-\frac{2}{16\pi^2}(3\lambda_T^2+2\lambda_D^2),\quad \Delta\gamma_S=0,\\
\Delta\beta_{\lambda_h} =&\frac{2\lambda_h^2}{16\pi^2}(3\lambda_T^2+2\lambda_D^2),\quad \Delta\beta_\lambda=\Delta\beta_\kappa=0,\\
\Delta\beta_{g_X^2} =&10\frac{2}{16\pi^2}Q_{f_1}^2g_X^4,\quad \Delta\beta_{g_i}=c_in\frac{g_i^4}{16\pi^2},
\end{aligned}\tag{B.3}$$

where $c=(5/3,1,1)$ and n is the pair of $(5,\bar{5})$ messengers. Using this result, we calculate the following soft terms at the messenger boundary

$$m_S^2=-\frac{\lambda_h^2}{(16\pi^2)^2}\left(3\lambda_T^2+2\lambda_D^2\right)\frac{F^2}{M^2},\tag{B.4}$$

$$A_{\lambda_h}=-\frac{1}{16\pi^2}\left(3\lambda_T^2+2\lambda_D^2\right)\frac{F}{M},\tag{B.5}$$

$$m_{H_h}^2=m_S^2+m_G^2,\tag{B.6}$$

$$\begin{aligned}
m_{\bar{H}_h}^2=&\frac{1}{(16\pi^2)^2}\Big[8\lambda_D^4+15\lambda_T^4+12\lambda_T^2\lambda_D^2-16g_3^2\lambda_T^2\\
&-6g_2^2\lambda_D^2-2g_1^2\left(\frac{2}{3}\lambda_T^2+\lambda_D^2\right)-2g_X^2(Q_{f_1}^2+Q_{H_h}^2)(3\lambda_T^2+2\lambda_D^2)\Big]\frac{F^2}{M^2}+m_G^2,
\end{aligned}\tag{B.7}$$

where $m_G^2=(Q_{H_h}/Q_{f_1})^2m_{\widetilde{X}}^2/(2n_X)$ (with dark gaugino mass $m_{\widetilde{X}}=2n_X\frac{Q_{f_1}^2g_X^2}{16\pi^2}\frac{F}{M}$ and $n_X=5$ in our paper) is the pure $U(1)_X$ GMSB contribution. And similar formula applies to the gauge mediation contribution to the soft mass term of other fields by replacing Q_{H_h} with corresponding charge.

C. One-Loop RGEs of Some Soft Terms

Here we present some important one-loop RGEs for all the soft terms in the dark sector, m_S^2, A_λ and A_κ in the NMSSM, and some Yukawa couplings and dark gauge couplings. In general, they take the form:

$$\frac{dY}{dt} = \frac{1}{16\pi^2}\beta_Y, \quad \frac{dA}{dt} = \frac{1}{16\pi^2}\beta_A, \quad \frac{dm^2}{dt} = \frac{1}{16\pi^2}\beta_{m^2}, \tag{C.1}$$

where $t \equiv \log(Q/Q_0)$ with Q_0 being the boundary energy scale and Q the running scale. Following a general calculation in [35], the RGEs for the soft terms in the dark sector are given by

$$\beta_{A_{\lambda_l}} = 9\lambda_l^2 A_{\lambda_l} + g_X^2 Q_{H_l}^2 \left(8m_{\widetilde{X}}\lambda_l - 4A_{\lambda_l}\right), \tag{C.2}$$

$$\beta_{m_H^2} = 2N\lambda_l^2\left(m_{H_l}^2 + m_{\bar{H}_l}^2 + m_H^2 + A_{\lambda_l}^2\right) - 8g_X^2 Q_H^2 m_{\widetilde{X}}^2 + 2Q_H g_X^2 \mathcal{S}_X, \tag{C.3}$$

$$\beta_{m_{H_l}^2} = 2\lambda_l^2\left(m_{H_1}^2 + m_{\bar{H}_1}^2 + m_H^2 + A_{\lambda_l}^2\right) - 8g_X^2 Q_{H_l}^2 m_{\widetilde{X}}^2 + 2Q_{H_l} g_X^2 \mathcal{S}_X, \tag{C.4}$$

$$\beta_{m_{\bar{H}_l}^2} = 2\lambda_l^2\left(m_{H_1}^2 + m_{\bar{H}_1}^2 + m_H^2 + A_{\lambda_l}^2\right) - 8g_X^2 Q_{\bar{H}_l}^2 m_{\widetilde{X}}^2 + 2Q_{\bar{H}_l} g_X^2 \mathcal{S}_X, \tag{C.5}$$

where $\mathcal{S}_X$ is defined in the text. For the modified NMSSM soft parameters, the new RGEs are given by

$$\begin{aligned}\beta_{m_S^2} &= 4\lambda^2\left(m_{H_u}^2 + m_{H_d}^2 + m_S^2 + A_\lambda^2\right) + 4\kappa^2\left(3m_S^2 + A_\kappa^2\right)\\ &\quad + 2\lambda_h^2\left(m_S^2 + m_{H_h}^2 + m_{\bar{H}_h}^2 + A_{\lambda_h}^2\right),\end{aligned} \tag{C.6}$$

$$\beta_{A_\lambda} = 8\lambda^2 A_\lambda + 6h_t^2 A_t + 6h_b^2 A_b + 2h_\tau^2 A_\tau + 4\kappa^2 A_\kappa + 2g_1^2 M_1 + 6g_2^2 M_2 + 2\lambda_h^2 A_{\lambda_h} \tag{C.7}$$

$$\beta_{A_\kappa} = 12\kappa^2 A_\kappa + 12\lambda^2 A_\lambda + 2\lambda_h^2 A_{\lambda_h}, \tag{C.8}$$

Finally, for the new Yukawa couplings and the dark gauge coupling, their RGEs are given by

$$\beta_{\lambda_h} = 2\lambda_h^2\left(2\kappa^2 + 3\lambda_h^2 + \lambda^2 - 4(Q_{H_h}g_X)^2\right) \quad (\text{below } M), \tag{C.9}$$

$$\beta_{\lambda_l} = 2\lambda_l^2\left(3\lambda_l^2 - 4(Q_{H_h}g_X)^2\right) \quad (\text{below } M), \tag{C.10}$$

$$\beta_{g_X^2} = 2g_X^4\left(10Q_{f_1}^2 + 2Q_{H_h}^2 + Q_{H_h}^2 + N(Q_{\bar{H}_l}^2 + Q_{H_l}^2)\right) \quad (\text{above } M), \tag{C.11}$$

$$\beta_{\xi_{1,T}} = \xi_{1,T}\left(2\xi_{1,T}^2 + \lambda_T^2 - \frac{16}{3}g_3^2 - \frac{9}{15}g_1^2\right) \quad (\text{above } M), \tag{C.12}$$

$$\beta_{\xi_{1,D}} = \xi_{1,D}\left(2\xi_{1,D}^2 + \lambda_D^2 - 3g_2^2 - \frac{4}{15}g_1^2\right) \quad (\text{above } M). \tag{C.13}$$

At the scale M_{GUT}, the unified value $\xi_{1,D} = \xi_{1,T}$ is assumed.

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