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Strong supersymmetric quantum effects on top quark production at the Fermilab Tevatron

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ABSTRACT

The supersymmetric QCD corrections to top quark pair production by $q\bar{q}$ annihilation in $p\bar{p}$ collisions are calculated in the minimal supersymmetric model. We consider effects of the mixing of the scalar top quarks on the corrections to the total $t\bar{t}$ production cross section at the Fermilab Tevatron. We found that such correction is less sensitive to squark mass and gluino mass than in no-mixing case, and in both cases the corrections can exceed 10% even if we consider the recent CDF limit on squark and gluino masses.

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The top quark has been discovered by CDF(D0) collaboration at Tevatron[1]. The mass and production cross section are found to be, respectively, $176 \pm 8(stat) \pm 10(syst)$ $(199^{+19}_{-21}(stat) \pm 22(syst))$GeV and $6.8^{+3.6}_{-2.4}(6.4 \pm 2.2)pb$ by CDF(D0) collaboration. At the Tevatron,the dominant production mechanism for a heavy top quark is the QCD process $q\bar{q} \to t\bar{t}$[2]. Recently, there has been a lot of interest in the one-loop radiative corrections of the top quark production cross section at the Tevatron, which arise from the new physics beyond the SM such as the two-Higgs-doublet model(2HDM) and the minimal supersymmetric model(MSSM)[4,5]. In Ref.[5], the supersymmetric QCD corrections to the top quark production in $p\bar{p}$ were calculated in the simplest case of unmixed squarks and degenerate masses. But we have to know the impact from the mixing squarks because of interest to phenomenology, especially, sensitive dependence of the supersymmetric QCD on the squark masses and gluino mass[5]. In that reference also only two cases of the gluino mass $m_{\tilde{g}} = 150\,\mathrm{GeV}$ and $m_{\tilde{g}} = 200\,\mathrm{GeV}$, respectively, was considered. The purpose of the present letter is to evaluate the supersymmetric QCD corrections to the top quark production at the Tevatron in the general case of the mixing squark masses and compare our results with that in the case of unmixed squark masses given in Ref.[5]. We also discuss further the dependence of such corrections on the gluino mass.

In the MSSM the strong supersymmetric interaction Lagrangian relevant to our calculation is given, in the present of squark mixing, by[6]

$$\mathcal{L}_{\tilde{g}\tilde{q}q} = -i\sqrt{2} g_s T^a (\bar{q} P_R \tilde{q}_L - \bar{q} P_L \tilde{q}_R)\tilde{g}_a + H.C., \tag{1}$$

where g_s is the strong coupling constant, T^a are $SU(3)_c$ generators, $P_{L,R} \equiv \frac{1}{2}(1 \pm \gamma_5)$, $\tilde{g}_a$ are the Majorana gluino fields, and $\tilde{q}_{L,R}$ are the current eigenstate squarks, which are related to the corresponding mass eigenstates $\tilde{q}_{1,2}$ by

$$\tilde{q}_1 = \tilde{q}_L \cos\theta + \tilde{q}_R \sin\theta, \quad \tilde{q}_2 = -\tilde{q}_L \sin\theta + \tilde{q}_R \cos\theta \tag{2}$$

The mixing angle θ as well as the masses $m_{\tilde{q}_{1,2}}$ of the physical squarks are determined by the following mass matrices[7]

$$\mathcal{M}_{\tilde{t}}^2 = \left\{ \begin{array}{cc} m_{\tilde{t}_L}^2 + m_t^2 + 0.35 D_Z & -m_t(A_t + \mu \cot\beta) \\ -m_t(A_t + \mu\cot\beta) & m_{\tilde{t}_R}^2 + m_t^2 + 0.16 D_Z \end{array} \right\}, \tag{3}$$

where $D_Z = M_Z^2 \cos 2\beta$ with $\tan\beta$ being the ratio of Higgs vcauum expectation values, $m_{\tilde{t}_L,\tilde{t}_R}$ are soft breaking masses, A_t is soft breaking parameter describing the strength of trilinear scalar interactions, and μ is the supersymmetric Higgs mass parameter. The relevant Feynman diagrams contributive to one-loop supersymmetric QCD corrections to $q\bar{q} \to t\bar{t}$ amplitiude are shown in Fig.1 of Ref.[5]. In our calculation we will follow the same notation and adopt the same regularization and renormalization scheme as was used in Ref.[5]. Notice that the off-diagonal elements of the squark mass matrices are proportional to the quark mass. In the case of supersymmetric partners of light quarks mixing between the current eigenstates can therefore be neglected. So we will take into account only the mixing between the squarks for the supersymmetric QCD corrections to the $gt\bar{t}$ coupling and neglect such mixing for the $gq\bar{q}$ couplings because these couplings mainly involved light quarks. The renormalized amlitude for $q\bar{q} \to t\bar{t}$ can be written as

$$M_{\rm ren} = M_0 + \delta M_1^{\rm vertex} + \delta M_2^{\rm vertex} + \delta M^{\rm box} \tag{4}$$

where M_0 is the amplitude at tree-level and δM represent the supersymmetric QCD corrections arising from the effective $gq\bar{q}(gt\bar{t})$ vertex and box diagrams, which are given by

$$\begin{aligned}
M_0 &= \bar{v}(p_2)(-ig_s T^a \gamma^\nu) u(p_1) \frac{-ig_{\mu\nu}}{\hat{s}} \bar{u}(p_3)(-ig_s T^a \gamma^\mu) v(p_4) && (5)\\
\delta M_1^{\rm vertex} &= \bar{v}(p_2)(-ig_s T^a \widehat{\Gamma}'^\nu) u(p_1) \frac{-ig_{\mu\nu}}{\hat{s}} \bar{u}(p_3)(-ig_s T^a \gamma^\mu) v(p_4) && (6)\\
\delta M_2^{\rm vertex} &= \bar{v}(p_2)(-ig_s T^a \gamma^\nu) u(p_1) \frac{-ig_{\mu\nu}}{\hat{s}} \bar{u}(p_3)(-ig_s T^a \widehat{\Gamma}^\mu) v(p_4) && (7)\\
\delta M^{\rm box} &= i\frac{7\alpha_s g_s^2}{48\pi} \{ \bar{u}(p_3) P_R u(p_1) \bar{v}(p_2) P_R v(p_4) f_1 \\
&\quad + \bar{u}(p_3) P_R u(p_1) \bar{v}(p_2) P_L v(p_4) f_2 + \bar{u}(p_3) P_L u(p_1) \bar{v}(p_2) P_R v(p_4) f_3
\end{aligned}$$

$$+\bar{u}(p_3)P_L u(p_1)\bar{v}(p_2)P_L v(p_4)f_4+\bar{u}(p_3)P_R u(p_1)\bar{v}(p_2)P_R \not{p}_3 v(p_4)f_5$$

$$+\bar{u}(p_3)P_R u(p_1)\bar{v}(p_2)P_L \not{p}_3 v(p_4)f_6+\bar{u}(p_3)P_L u(p_1)\bar{v}(p_2)P_R \not{p}_3 v(p_4)f_7$$

$$+\bar{u}(p_3)P_L u(p_1)\bar{v}(p_2)P_L \not{p}_3 v(p_4)f_8+\bar{u}(p_3)\not{p}_4 P_R u(p_1)\bar{v}(p_2)P_R v(p_4)f_9$$

$$+\bar{u}(p_3)\not{p}_4 P_R u(p_1)\bar{v}(p_2)P_L v(p_4)f_{10}+\bar{u}(p_3)\not{p}_4 P_L u(p_1)\bar{v}(p_2)P_R v(p_4)f_{11}$$

$$+\bar{u}(p_3)\not{p}_4 P_L u(p_1)\bar{v}(p_2)P_L v(p_4)f_{12}+\bar{u}(p_3)\not{p}_4 P_R u(p_1)\bar{v}(p_2)P_R \not{p}_3 v(p_4)f_{13}$$

$$+\bar{u}(p_3)\not{p}_4 P_R u(p_1)\bar{v}(p_2)P_L \not{p}_3 v(p_4)f_{14}+\bar{u}(p_3)\not{p}_4 P_L u(p_1)\bar{v}(p_2)P_R \not{p}_3 v(p_4)f_{15}$$

$$+\bar{u}(p_3)\not{p}_4 P_L u(p_1)\bar{v}(p_2)P_L \not{p}_3 v(p_4)f_{16}+\bar{u}(p_3)\gamma_\mu P_R u(p_1)\bar{v}(p_2)P_R\gamma^\mu v(p_4)f_{17}$$

$$+\bar{u}(p_3)\gamma_\mu P_R u(p_1)\bar{v}(p_2)P_L\gamma^\mu v(p_4)f_{18}+\bar{u}(p_3)\gamma_\mu P_L u(p_1)\bar{v}(p_2)P_R\gamma^\mu v(p_4)f_{19}$$

$$+\bar{u}(p_3)\gamma_\mu P_L u(p_1)\bar{v}(p_2)P_L\gamma^\mu v(p_4)f_{20}\} \tag{8}$$

with

$$\widehat{\Gamma}'^{\nu} = \gamma^\nu F_1^{'}+\gamma^\nu\gamma_5 F_2^{'}+k^\nu F_3^{'}+k^\nu\gamma_5 F_4^{'}+ik_\mu\sigma^{\nu\mu}F_5^{'}+ik_\mu\sigma^{\nu\mu}\gamma_5 F_6^{'} \tag{9}$$

$$\widehat{\Gamma}^{\mu} = \gamma^\mu F_1+\gamma^\mu\gamma_5 F_2+k^\mu F_3+k^\mu\gamma_5 F_4+ik_\nu\sigma^{\mu\nu}F_5+ik_\nu\sigma^{\mu\nu}\gamma_5 F_6 \tag{10}$$

Here,$k=p_1+p_2$,p_1,p_2 denote the momenta of incoming partons,and p_3,p_4 are used for outgoing top-quark and its antiparticle. $F_i^{'},F_i$ and f_i are form factors which are presented in Appendix.

The renormalized differential cross section of the subprocess is given by

$$\frac{d\hat{\sigma}}{d\cos\theta}=\frac{1}{32\pi}\frac{\beta_t}{\hat{s}}|M_{\rm ren}|^2, \tag{11}$$

where

$$|M_{\rm ren}|^2=\overline{\sum}|M_0|^2+2{\rm Re}\overline{\sum}M_0^+(\delta M_1^V+\delta M_2^V+\delta M^b), \tag{12}$$

with

$$\overline{\sum}|M_0|^2=\frac{4g_s^4}{9\hat{s}^2}[2\hat{s}m_t^2+(\hat{t}-m_t^2)^2+(\hat{u}-m_t^2)^2], \tag{13}$$

$$\overline{\sum}M_0^+\delta M_1^V=\frac{4g_s^4}{9\hat{s}^2}[2\hat{s}m_t^2+(\hat{t}-m_t^2)^2+(\hat{u}-m_t^2)^2](F_1^{'}-1), \tag{14}$$

$$\overline{\sum}M_0^+\delta M_2^V=\frac{4g_s^4}{9\hat{s}^2}\{[2\hat{s}m_t^2+(\hat{t}-m_t^2)^2+(\hat{u}-m_t^2)^2](F_1-1)+2m_t^2\hat{s}^2F_5\}, \tag{15}$$

$$\overline{\sum} M_0^+ \delta M^b = \frac{7g_s^4}{216\hat{s}}\frac{\alpha_s}{\pi}\{[\hat{s}m_t^2 + (\hat{t}-m_t^2)^2](f_2+f_3)$$
$$-m_t[\hat{s}^2+\hat{s}\hat{t}-2m_t^2\hat{s}-(\hat{t}-m_t^2)(\hat{s}+\hat{t}-m_t^2)](f_6+f_7-f_{10}-f_{11})$$
$$-[m_t^4\hat{s}+(\hat{s}-m_t^2)(\hat{s}+\hat{t}-m_t^2)^2+2m_t^2(\hat{t}-m_t^2)$$
$$(\hat{s}+\hat{t}-m_t^2)](f_{14}+f_{15})-2[m_t^2\hat{s}+(\hat{s}+\hat{t}-m_t^2)^2](f_{18}+f_{19})\}, \quad (16)$$

In the above equations, θ is the scattering angle between the quark and the top quark, $\hat{s}$, $\hat{t}$, and $\hat{u}$ are the kinematic invariants for the $2 \to 2$ subprocess with $\hat{s}+\hat{t}+\hat{u}=2m_t^2$, and $\beta_t = \sqrt{1-4m_t^2/\hat{s}}$ the velocity of the final quarks. The total cross section for the production of top quark pair can be written in the form

$$\sigma(s) = \sum_{i,j}\int dx_1 dx_2 \widehat{\sigma}_{ij}(x_1x_2s, m_t^2, \mu^2)[F_i^A(x_1,\mu)F_j^B(x_2,\mu)+(A\leftrightarrow B)], \quad (17)$$

with

$$\begin{array}{ll} s=(P_1+P_2)^2, & \widehat{s}=x_1x_2s, \\ p_1=x_1P_1, & p_2=x_2P_2, \end{array} \quad (18)$$

where A and B denote the incident hadrons, P_1 and P_2 are their four-momenta, i, j are the initial partons, x_1 and x_2 are their longitudinal momentum fractions, and the functions F_i^A, F_j^B are the parton distributions of the initial-state hadrons A and B. In our numerical calculations, we have used the MRS Set A' parton distribution functions [8], and do not consider SUSY corrections to the parton distribution functions since the principle of decoupling demands that the corrections are negligible. Introducing a convenient variable $\tau = x_1x_2$ and changing to x_1 and τ as independent variables, the total cross section expression becomes

$$\sigma(s) = \sum_{i,j}\int_{\tau_0}^{1}\frac{d\tau}{\tau}\left(\frac{1}{s}\frac{dL_{ij}}{d\tau}\right)(\widehat{s}\widehat{\sigma_{ij}}), \quad (19)$$

where $\tau_0 = 4m_t/s$ and the quantity $dL_{ij}/d\tau$ is the parton luminosity, which is defined as

$$\frac{dL_{ij}}{d\tau} = \int_{\tau}^{1}\frac{dx_1}{x_1}[F_i^A(x_1,\mu), F_j^B(\frac{\tau}{x_1},\mu)+(A\leftrightarrow B)]. \quad (20)$$

Now we present some numerical examples. In our numerical calculation, we input $m_t = 176$ GeV and 1-loop $\alpha_s(Q^2 = \hat{s})$ and use the phase space cuts: $|\eta| < 2.5, \quad p_T > 20 GeV$. As to the supersymmetric parameters involved in our calculations, in general, once $\tan\beta$ and $m_{\tilde{t}_L}$ are fixed, we are free to choose two independent parameters in the stop mass matrix : $(m_{\tilde{t}_R}, A_t + \mu\cot\beta)$, which also can be tranfered to $(m_{\tilde{t}_R}, m_{\tilde{t}_1})$. As explained above, we neglect the mixing of squark masses in the corrections to the $gq\bar{q}$ couplings and also neglect the mass splittings between squarks of different flavor for simplicity.

In Figs.1-3 we give some numerical results in a simple case, in which we set $\tan\beta = 1$ and $m_{\tilde{t}_L} = m_{\tilde{t}_R} = m_{\tilde{t}_1} = m_{\tilde{q}}$ and thus the mixing angle is $\pi/4$ and $A_t + \mu = m_t$. To compare the results in no-mixing case with those in the mixing case, we show the results in both cases in Figs.1-3. From these figures we can see that the corrections in mixing case are smaller than in the no-mixing case. The plots versus squark and gluino masses in the mixing case get smoother than in no-mixing case. Fig.1 show the dependence of the corrections on gluino mass for fixed squark mass of 150 GeV. The corrections can be either positive or negative, depending on the gluino mass. The corrections are negative for gluino mass below 150 GeV and above that they become positive. When gluino mass is changed from 100 GeV to 200 GeV, the corrections vary from -2%(-10%) to 16%(23%) in mixing (no-mixing) case. The corrections get their positive maximum size at gluino mass of 200 GeV. When gluino mass is larger than 200 GeV, the corrections in both cases drop monotonously with the increase of gluino mass and tend to zero at 600 GeV, showing the decoupling effects. The recent CDF lower limit [9] on gluino mass is 160 GeV for arbitrary squark mass and 220 GeV when gluino mass is equal to squark mass. So the corrections are always positive if we consider the CDF limit on gluino mass. Fig.2 and Fig.3 show the dependence of the corrections on squark mass for fixed gluino masses of 150 GeV and 200 GeV, respectively. The corrections in the mixing case differ significantly from those in no-mixing case for low squark mass. For gluino mass of 200 GeV the CDF lower limit [9] for squark mass is about 220 GeV. At this lower limit the corrections are 14% and 18% for the mixing and no-mixing cases, respectively. But with the increase of squark mass the difference of the corrections in both cases is getting negligibly small.

From Fig1-3 we have found that the corrections vary rapidly with gluino mass, especially for 150 Gev $\leq m_{\tilde{g}} \leq$ 200 GeV, though mixied cases are smaller than unmixied ones. This is due to the fact that we have set $m_t = 176$GeV in the numerical calculation, and the threshold for open top quark production is crossed in that region. If we change the top quark mass, we can find that such region is also shifted correspondingly, which provides a check on our calculation, especially on the treatment of the threshold.

We also perform the numerical calculations for $\tan\beta = 10$. We found that the corrections are not sensitive to $\tan\beta$ value. For example, with gluino mass of 200 GeV and squark mass of 150 Gev we get $\Delta\sigma = 0.482\,pb$ for $\tan\beta = 1$ and $\Delta\sigma = 0.490\,pb$ for $\tan\beta = 10$.

In conclusion, we have shown that the supersymmetric QCD corrections to top quark pair production by $q\bar{q}$ annihilation in $p\bar{p}$ collisions can exceed 10% in both the mixing and no-mixing cases of squark masses even if we consider the recent CDF limit on squark and gluino masses.

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Appendix

The form factor F_1 is given by

$$F_1 = 1 + \frac{\alpha_s}{3\pi}\left[A_{ii}\left(F_1^{(t\tilde{g}\tilde{t}_i)} + 2m_t^2 G_1^{(t\tilde{g}\tilde{t}_i)}\right)\right.$$

$$+2m_t m_{\tilde{g}} B_{ii} G_0^{(t\tilde{g}\tilde{t}_i)} + \frac{1}{8}(F_1^{(1)} + 9F_1^{(2)})\Big], \tag{21}$$

where

$$A_{ii} = a_i a_i + b_i b_i, \quad B_{ii} = a_i a_i - b_i b_i, \tag{22}$$

$$a_1 = -b_2 = \frac{1}{\sqrt{2}}(\cos\theta - \sin\theta), a_2 = b_1 = -\frac{1}{\sqrt{2}}(\cos\theta + \sin\theta), \tag{23}$$

$$F_1^{(t\tilde{g}\tilde{t}_i)} = \int_0^1 dy\ y \ln\left[\frac{-m_t^2 y(1-y) + m_{\tilde{g}}^2(1-y) + m_{\tilde{t}_i}^2 y}{\mu^2}\right], \tag{24}$$

$$G_n^{(t\tilde{g}\tilde{t}_i)} = -\int_0^1 dy\ \frac{y^{n+1}(1-y)}{-m_t^2 y(1-y) + m_{\tilde{g}}^2(1-y) + m_{\tilde{t}_i}^2 y}, \tag{25}$$

$$F_1^{(1)} = 2m_t[m_{\tilde{g}} B_{ii}(c_0 + c_{11}) - m_t A_{ii}(c_{11} + c_{21})] - 2A_{ii}\bar{c}_{24}(-p_3, k, m_{\tilde{g}}, m_{\tilde{t}_i}, m_{\tilde{t}_i}), \tag{26}$$

$$F_1^{(2)} = -1 + A_{ii}[2\bar{c}_{24} + \hat{s}(c_{22} - c_{23}) - m_{\tilde{g}}^2 c_0 - m_t^2(c_0 + 2c_{11} + c_{21})](-p_3, k, m_{\tilde{t}_i}, m_{\tilde{g}}, m_{\tilde{g}})$$

$$-2B_{ii} m_t m_{\tilde{g}}(c_0 + c_{11})(-p_3, k, m_{\tilde{t}_i}, m_{\tilde{g}}, m_{\tilde{g}}), \tag{27}$$

Here θ and θ' are the mixing angels of squarks. c_0, c_{ij} are the 3-point Feynman integrals, definition and expression for which can be found in Ref.[10]. The form factor F_1' is obtained by

$$F_1' = 1 + \frac{\alpha_s}{3\pi}\left[K_{jj} F_1^{(q\tilde{g}\tilde{q}_j)} + \frac{1}{8}(F_1^{(1)'} + 9F_1^{(2)'})\right], \tag{28}$$

where

$$K_{jj} = A_{jj}|_{\theta\to\theta'}, L_{jj} = B_{jj}|_{\theta\to\theta'}, \tag{29}$$

$$F_1^{(1)'} = -2K_{jj}\bar{c}_{24}(p_1, -k, m_{\tilde{g}}, m_{\tilde{q}_j}, m_{\tilde{q}_j}), \tag{30}$$

$$F_1^{(2)'} = -1 + K_{jj}[2\bar{c}_{24} + \hat{s}(c_{22} - c_{23}) - m_{\tilde{g}}^2 c_0](p_1, -k, m_{\tilde{q}_j}, m_{\tilde{g}}, m_{\tilde{g}}) \tag{31}$$

F_5 is given by

$$F_5 = \frac{\alpha_s}{24\pi}\left[F_5^{(1)} + 9F_5^{(2)}\right], \tag{32}$$

where

$$F_5^{(1)} = [m_t A_{ii}(c_{11} + c_{21}) - m_{\tilde{g}} B_{ii}(c_0 + c_{11})](-p_3, k, m_{\tilde{g}}, m_{\tilde{t}_i}, m_{\tilde{t}_i}), \tag{33}$$

$$F_5^{(2)} = [m_t A_{ii}(c_{11} + c_{21}) + m_{\tilde{g}} B_{ii} c_{11}](-p_3, k, m_{\tilde{t}_i}, m_{\tilde{g}}, m_{\tilde{g}}), \tag{34}$$

f_i are given by

$$f_2 = m_{\tilde{g}}^2 \sigma_{ij}'^2 D_0 - m_t m_{\tilde{g}} \sigma_{ij} \sigma_{ij}' (D_{12} + D_{13} - D_0 - D_{11}) + m_t^2 \sigma_{ij}^2 (D_{26} - D_{12} - D_{24}), \quad (35)$$

$$f_6 = -m_{\tilde{g}} \sigma_{ij} \sigma_{ij}' (D_0 + D_{11}) + m_t \sigma_{ij}^2 (D_{12} + D_{24}), \quad (36)$$

$$f_{10} = m_{\tilde{g}} \sigma_{ij} \sigma_{ij}' (D_{13} - D_{12}) - m_t \sigma_{ij}^2 (D_{12} + D_{23} + D_{24} - D_{13} - D_{26} - D_{25}), \quad (37)$$

$$f_{14} = \sigma_{ij}^2 (D_{12} + D_{24} - D_{13} - D_{25}), \quad (38)$$

$$f_{18} = \sigma_{ij}^2 D_{27}, \quad (39)$$

$$(f_3, f_7, f_{11}, f_{15}, f_{19}) = (f_2, f_6, f_{10}, f_{14}, f_{18}) \Big|_{\sigma_{ij} \to \lambda_{ij}} \quad (40)$$

where

$$\sigma_{ij} = (a_i + b_i)(c_j + d_j), \sigma_{ij}' = (a_i - b_i)(c_j + d_j),$$

$$\lambda_{ij} = (a_i - b_i)(c_j - d_j), \lambda_{ij}' = (a_i + b_i)(c_j - d_j),$$

$$(c_j, d_j) = (a_j, b_j)|_{\theta \to \theta')} \quad (41)$$

Here $D_0, D_{ij}(-p_1, -p_2, p_4, m_{\tilde{g}}, m_{\tilde{q}_j}, m_{\tilde{g}}, m_{\tilde{t}_i})$ are the 4-point Feynman integrals [10].