

# Studies of Charmonium at BESIII

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**Abstract.** In recent years, lots of studies of charmonium decays have been performed at BESIII based on large data samples of  $J/\psi$ ,  $\psi(3686)$  and  $\psi(3770)$ . Recent results in searches for radiative transitions of  $\psi(3770)$  and rare phenomena in charmonium decays, and studies of light hadrons structures and properties will be presented.

## INTRODUCTION

The BESIII detector at the BEPCII collider has accumulated the world's largest data samples of  $e^+e^-$  collisions in the  $\tau$ -charm region, including  $J/\psi$ ,  $\psi(3686)$ ,  $\psi(3770)$  and the XYZ data. These data samples provide a clean and simple environment to study the production and decay mechanisms of charmonium states including not only  $J/\psi$ ,  $\psi(3686)$ ,  $\psi(3770)$ , but also  $\eta_c$ ,  $\eta_c(2S)$  and  $\chi_{cJ}$  which are available by the  $\gamma$  transition of  $\psi(3686)$  and  $\psi(3770)$ .

Results in this presentation are based on  $2.92 \text{ fb}^{-1}$  data sample taken at  $\sqrt{s} = 3.773 \text{ GeV}$ ,  $1.06 \times 10^8 \psi(3686)$  events and  $1.31 \times 10^9 J/\psi$  events.

## SEARCH FOR $\psi(3770)$ RADIATIVE TRANSITIONS

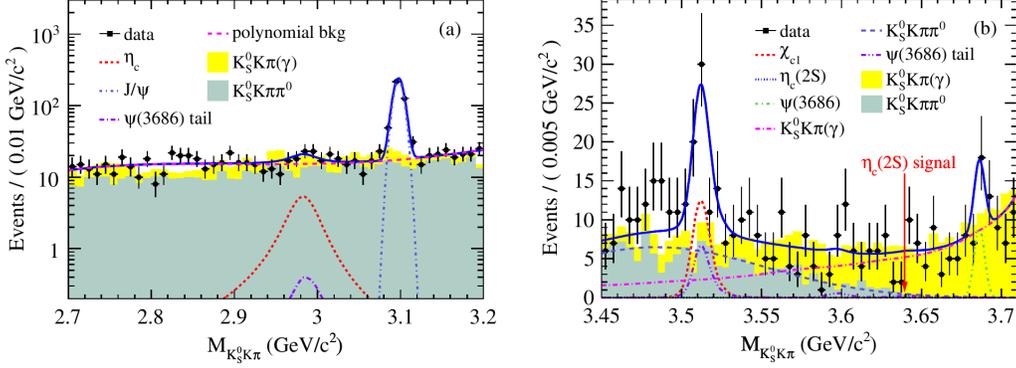
The nature of the excited  $J^{PC} = 1^{--} c\bar{c}$  bound states above the  $D\bar{D}$  threshold is of interest but still not well known. The  $\psi(3770)$  resonance, as the lightest charmonium state lying above the open charm threshold, is generally assigned to be a dominant  $1^3D_1$  momentum eigenstate with a small  $2^3S_1$  admixture [1]. It has been thought almost entirely to decay to  $D\bar{D}$  states [2, 3]. Unexpectedly, the BES Collaboration found a large inclusive non- $D\bar{D}$  branching fraction,  $(14.7 \pm 3.2)\%$ , by utilizing various methods [4, 5, 6, 7]. A later work by the CLEO Collaboration found a contradictory non- $D\bar{D}$  branching fraction,  $(3.3 \pm 1.4^{+6.6}_{-4.8})\%$  [8]. The BES results suggest substantial non- $D\bar{D}$  decays, although the CLEO result finds otherwise. In the exclusive analyses, various non- $D\bar{D}$  decay modes have been observed, including hadronic transitions  $\psi(3770) \rightarrow J/\psi\pi^+\pi^-$  [9, 10],  $\pi^0\pi^0 J/\psi$ ,  $\eta J/\psi$  [10], the  $E1$  transitions  $\gamma\chi_{cJ}$  ( $J = 0, 1$ ) [11, 12], and the decay to light hadrons  $\phi\eta$  [13]. The sum of the observed non- $D\bar{D}$  exclusive components still makes up less than 2% of all decays [14], which motivates the search for other exclusive non- $D\bar{D}$  final states.

### Search for $\psi(3770) \rightarrow \gamma\eta_c$ and $\gamma\eta_c(2S)$

The radiative transitions  $\psi(3770) \rightarrow \gamma\eta_c(\eta_c(2S))$  are supposed to be highly suppressed by selection rules, considering the  $\psi(3770)$  is predominantly the  $1^3D_1$  state. However, due to the non-vanishing photon energy in the decay, higher multipoles beyond the leading one could contribute [15]. Recently, the partial decay widths  $\Gamma(\psi(3770) \rightarrow \gamma\eta_c(\eta_c(2S)))$  have been calculated in Ref. [15] by considering contributions from the intermediate meson loop (IML) mechanism.

Using the  $\psi(3770)$  data sample, the radiative transitions  $\psi(3770) \rightarrow \gamma\eta_c(\eta_c(2S))$  through the decay process  $\psi(3770) \rightarrow K_S^0 K^\pm \pi^\mp$  have been searched for [16]. Figure 1 shows the invariant-mass spectrum of  $K_S^0 K^\pm \pi^\mp$  for selected candidates, together with the estimated backgrounds in the  $\eta_c$  mass region (Fig. 1(a)) and in the  $\chi_{c1} - \eta_c(2S)$  mass region (Fig. 1(b)). No significant  $\eta_c$  and  $\eta_c(2S)$  signals are observed. The upper limits on the branching fractions at a 90% C.L. have been set:  $\mathcal{B}(\psi(3770) \rightarrow \gamma\eta_c) < 6.8 \times 10^{-4}$  and  $\mathcal{B}(\psi(3770) \rightarrow \gamma\eta_c(2S)) < 2.0 \times 10^{-3}$ . The upper limit for  $\Gamma(\psi(3770) \rightarrow \gamma\eta_c)$  is within the error range of the theoretical predictions of IML [15] and lattice QCD

calculations [17]. However, the upper limit for  $\Gamma(\psi(3770) \rightarrow \gamma\eta_c(2S))$  is much larger than the prediction and is limited by statistics and the systematic error.

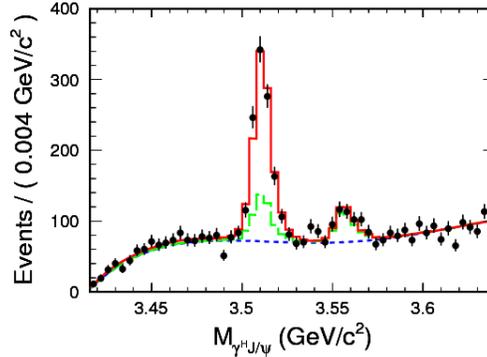


**FIGURE 1.** Invariant-mass spectrum for  $K_S^0 K^+ \pi^-$  from data with the estimated backgrounds and best-fit results superimposed in the (a)  $\eta_c$  and (b)  $\chi_{c1}$  and  $\eta_c(2S)$  mass regions. Dots with error bars are data. The shaded histograms represent the background contributions and the solid lines show the total fit results.

### Measurement of the Branching Fraction for $\psi(3770) \rightarrow \gamma\chi_{cJ}$

Within an S-D mixing model [1], Refs. [18, 19, 20] predict the partial widths for  $\psi(3770)$   $E1$  radiative transitions, but with large uncertainties. Up to now, the transition  $\psi(3770) \rightarrow \gamma\chi_{c2}$  has not been observed. Precision measurements of partial widths of the  $\psi(3770) \rightarrow \gamma\chi_{c1,2}$  processes are critical to test the theoretical predictions, and to better understand the nature of the partial widths of the  $\psi(3770)$ , as well as to find the origin of the non- $D\bar{D}$  decays of  $\psi(3770)$ .

Using the  $\psi(3770)$  data sample, the  $\psi(3770)$   $E1$  transitions  $\psi(3770) \rightarrow \gamma\chi_{c1,2}$  have been studied [21] by reconstructing  $\chi_{cJ}$  using the decay  $\chi_{cJ} \rightarrow \gamma J/\psi$ . Figure 2 shows the invariant-mass distribution of the higher energetic photon and  $J/\psi$ . Clear peak corresponding to the  $\chi_{c1}$  signal is observed while there is no significant signal of  $\chi_{c2}$ . The branching fraction of  $\psi(3770) \rightarrow \gamma\chi_{c1}$  is measured to be  $\mathcal{B}(\psi(3770) \rightarrow \gamma\chi_{c1}) = (2.48 \pm 0.15 \pm 0.23) \times 10^{-3}$ , which is the most precise measurement to date. The upper limit on the branching fraction of  $\psi(3770) \rightarrow \gamma\chi_{c2}$  at 90% C.L. is  $\mathcal{B}(\psi(3770) \rightarrow \gamma\chi_{c2}) < 0.64 \times 10^{-3}$ .



**FIGURE 2.** Invariant mass spectrum of higher energetic photon and  $J/\psi$  selected from data. The dots with error bars represent the data. The solid (red) line shows the fit. The dashed (blue) line shows the smooth background. The long-dashed (green) line is the sum of the smooth background and the contribution from  $e^+e^- \rightarrow (\gamma_{ISR})\psi(3686)$  production.

# SEARCH FOR RARE PHENOMENA IN CHARMONIUM DECAYS

## Search for Isospin-violating Transition $\chi_{c0,2} \rightarrow \pi^0 \eta_c$

Based upon an effective-field theoretical approach, theoretical calculations give qualitative insights in the isospin-breaking mechanisms in charmonium decays below  $D\bar{D}$  threshold [22]. Currently, for such a theory, quantitative predictions of individual branching fractions of isospin-forbidden decays of charmonium require more constraints from experimental data.

Using the  $\psi(3686)$  data sample, the hadronic isospin-violating transitions  $\chi_{c0,2} \rightarrow \pi^0 \eta_c$  have been searched for through  $\eta_c \rightarrow K_S^0 K^\pm \pi^\mp$  decays [23]. No statistically significant signal is observed and upper limits on the branching fractions for the processes  $\chi_{c0,2} \rightarrow \pi^0 \eta_c$  have been obtained. The results are  $\mathcal{B}(\chi_{c0} \rightarrow \pi^0 \eta_c) < 1.6 \times 10^{-3}$  and  $\mathcal{B}(\chi_{c2} \rightarrow \pi^0 \eta_c) < 3.2 \times 10^{-3}$ . These are the first upper limits on  $\mathcal{B}(\chi_{c0,2} \rightarrow \pi^0 \eta_c)$  that have been reported so far. These limits might help to constrain non-relativistic field theories and provide insight in the role of charmed-meson loops to the various transitions in charmonium and charmonium-like states.

## Search for $C$ -parity Violation in $J/\psi \rightarrow \gamma\gamma, \gamma\phi$

In the Standard Model (SM),  $C$  invariance is held in strong and electromagnetic (EM) interactions. Until now, no  $C$ -violating processes have been observed in EM interactions. Any evidence for  $C$  violation in the EM sector would immediately indicate physics beyond the SM.

Using the  $\psi(3686)$  data sample, the decays of  $J/\psi \rightarrow \gamma\gamma$  and  $\gamma\phi$  have been searched for via  $\psi(3686) \rightarrow J/\psi \pi^+ \pi^-$  [24]. No significant signal is observed for  $J/\psi \rightarrow \gamma\gamma$  and  $J/\psi \rightarrow \gamma\phi$ . The upper limits for the branching fractions of  $J/\psi \rightarrow \gamma\gamma$  and  $J/\psi \rightarrow \gamma\phi$  are set to be  $\mathcal{B}(J/\psi \rightarrow \gamma\gamma) < 2.7 \times 10^{-7}$  and  $\mathcal{B}(J/\psi \rightarrow \gamma\phi) < 1.4 \times 10^{-6}$ , respectively. The upper limit on  $\mathcal{B}(J/\psi \rightarrow \gamma\gamma)$  is one of magnitude more stringent than the previous upper limit [25], and  $\mathcal{B}(J/\psi \rightarrow \gamma\phi)$  is the first upper limit for this channel.

## Observation of OZI-suppressed Decay $J/\psi \rightarrow \pi^0 \phi$

A full investigation of  $J/\psi$  decaying to a vector meson ( $V$ ) and a pseudoscalar meson ( $P$ ) can provide rich information about SU(3) flavor symmetry and its breaking, probe the quark and gluon content of the pseudoscalar mesons, and determine the electromagnetic amplitudes [26, 27, 28].

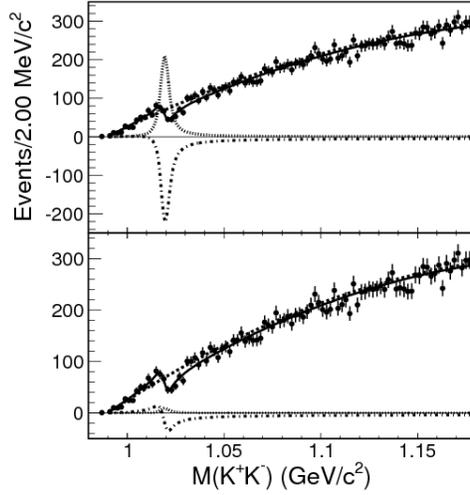
Using the  $J/\psi$  data sample, the first evidence for a doubly OZI suppressed electromagnetic  $J/\psi$  decay  $J/\psi \rightarrow \pi^0 \phi \rightarrow K^+ K^- \gamma\gamma$  has been reported [29]. A clear structure in the  $K^+ K^-$  invariant mass spectrum around 1.02 GeV/ $c^2$  is observed, which can be attributed to interference of  $J/\psi \rightarrow \pi^0 \phi$  and  $J/\psi \rightarrow K^+ K^- \pi^0$  decays. Figure 3 shows the fit to the invariant mass spectrum of  $K^+ K^-$  with the background events estimated with  $\pi^0$  sidebands subtracted. Due to the interference, two possible solutions are found. The corresponding measured values of the branching fraction of  $J/\psi \rightarrow \pi^0 \phi$  are  $(2.94 \pm 0.16 \pm 0.16) \times 10^{-6}$  and  $(1.24 \pm 0.33 \pm 0.30) \times 10^{-7}$ , respectively.

# STUDIES OF LIGHT HADRON STRUCTURES AND PROPERTIES

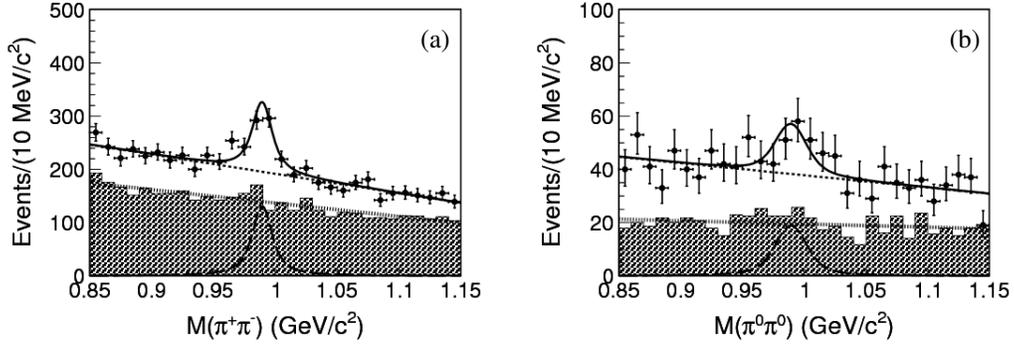
## Study of $J/\psi \rightarrow \phi \pi^0 f_0(980)$

The nature of the scalar meson  $f_0(980)$  is a long-standing puzzle [14]. In the study of  $J/\psi$  radiatively decaying into  $\pi^+ \pi^- \pi^0$  and  $\pi^0 \pi^0 \pi^0$ , BESIII observed the decay of  $\eta(1405) \rightarrow \pi^0 f_0(980)$  with a large isospin violation and an anomalously narrow width of  $f_0(980)$  [30]. One proposed explanation for these phenomena is the triangle singularity mechanism [31, 32].

Using the  $J/\psi$  data sample, the decays  $J/\psi \rightarrow \phi \pi^0 \pi^0 \pi^0$  with  $\phi \rightarrow K^+ K^-$  are investigated [33]. The isospin-violating decay  $J/\psi \rightarrow \phi \pi^0 f_0(980)$  is observed for the first time. Figure 4 shows the invariant mass spectrum of  $\pi^+ \pi^-$  (Fig. 4(a)) and  $\pi^0 \pi^0$  (Fig. 4(b)). A clear  $f_0(980)$  exists for the  $\pi^+ \pi^-$  mode. The width obtained from the dipion mass spectrum is  $(15.3 \pm 4.7)$  MeV/ $c^2$ , which is consistent with that in the study of  $J/\psi \rightarrow \gamma \eta(1405) \rightarrow \gamma \pi^0 f_0(980)$  [30] and the prediction of a theoretical work [34] based on the triangle singularity mechanism [31, 32]. In the invariant mass spectra of  $f_0(980) \pi^0$ , there is evidence of a resonance around 1.28 GeV/ $c^2$  for the  $f_0(980) \rightarrow \pi^+ \pi^-$  mode, which was identified as the axial-vector meson  $f_1(1285)$ .



**FIGURE 3.** Fit to  $M(K^+K^-)$  spectrum after sideband subtraction for Solution I (a) and Solution II (b). The red dotted curve denotes the  $\phi$  resonance; the blue dashed curve is the non- $\phi$  contribution; the green dot-dashed curve represents their interference; the blue solid curve is the sum of them.



**FIGURE 4.** The spectra (a)  $M(\pi^+\pi^-)$  and (b)  $M(\pi^0\pi^0)$  (three entries per event) with  $K^+K^-$  in the  $\phi$  signal region (black dots) and in the  $\phi$  sideband regions (hatched histogram). The solid curve is the full fit, the long-dashed curve is the  $f_0(980)$  signal, the dotted line is the non- $\phi$  background, and the short-dashed line is the total background.

### Study of $\chi_{cJ} \rightarrow \eta' K^+ K^-$

Until now,  $K_0^*(1430)$  has been observed in  $K_0^*(1430) \rightarrow K\pi$  only, but it is also expected to couple to  $\eta'K$  [35, 36].  $\chi_{c1} \rightarrow \eta'K^+K^-$  is a promising channel to search for  $K_0^*(1430)$  and study its properties while decays of  $\chi_{c0,2} \rightarrow K_0^*(1430)K$  are forbidden by spin-parity conservation.

Using the  $\psi(3686)$  data sample, the decay  $\chi_{cJ} \rightarrow \eta'K^+K^-$  with  $\eta' \rightarrow \gamma\rho^0$  and  $\eta' \rightarrow \eta\pi^+\pi^-$ ,  $\eta \rightarrow \gamma\gamma$  is studied for the first time [37]. Abundant structures on the  $K^+K^-$  and  $\eta K^\pm$  invariant mass spectra are observed for  $\chi_{c1}$  candidate events, and a partial wave analysis is performed for the decay  $\chi_{c1} \rightarrow \eta'K^+K^-$ . Figure 5 shows the comparisons of data and fit projections in terms of the invariant mass spectra of  $\eta'K^\pm$  for  $\chi_{c1}$  candidate events. The partial branching fractions of  $\chi_{c1}$  decay processes with intermediate states  $f_0(980)$ ,  $f_0(1710)$ ,  $f_2'(1525)$  and  $K_0^*(1430)$  are measured for the first time.

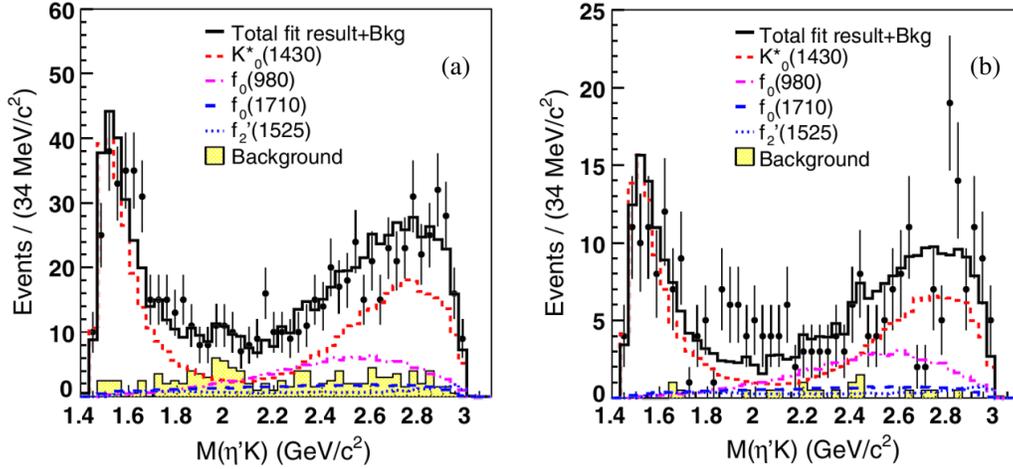


FIGURE 5. The invariant mass distribution of  $\eta'K^\pm$  within the  $\chi_{c1}$  mass range for (a)  $\eta' \rightarrow \gamma\rho^0$  mode and (b)  $\eta' \rightarrow \eta\pi^+\pi^-$  mode.

### Study of $\chi_{cJ}$ Decaying into $\phi K^*(892)\bar{K}$

The nature of the axial-vector candidate,  $h_1(1380)$  is still controversial [38, 39]. The direct observation of the  $h_1(1380)$  in experiments and the precise measurement of its resonance parameters may shed light on its nature and aid in identifying the ground state axial-vector meson nonet in the quark model.

Using the  $\psi(3686)$  data sample, the first measurement of  $\chi_{cJ} \rightarrow \phi K_S^0 K^\pm \pi^\mp$  and  $\chi_{cJ} \rightarrow \phi K^+ K^- \pi^0$  has been reported [40]. The decays are dominated by the three-body reaction  $\chi_{cJ}$  decaying into  $\phi K^*(892)\bar{K}$ . The branching fractions for this reaction via neutral and charged  $K^*(892)$  are measured for the first time. Figure 6 shows the invariant mass spectrum of  $K\bar{K}\pi$ , a significant excess of events above the phase space expectation is observed near the  $K^*(892)\bar{K}$  mass threshold in the decays of  $\chi_{c1,2}$  with a significance greater than  $10\sigma$ . The observed structure has negative  $C$  parity, and is expected to be the  $h_1(1380)$  state, considering its mass, width and decay through  $K^*(892)\bar{K}$ . The mass and width of the  $h_1(1380)$  are determined to  $(1412 \pm 4 \pm 8)$  MeV/ $c^2$  and  $(84 \pm 12 \pm 40)$  MeV, respectively. This is the first evidence of the  $h_1(1380)$  in its decay to  $K^*(892)\bar{K}$ . Evidence is also found for the decays  $\chi_{cJ} \rightarrow \phi\phi(1680)$  and  $\chi_{cJ} \rightarrow \phi\phi(1850)$ , but with significances less than  $5\sigma$ .

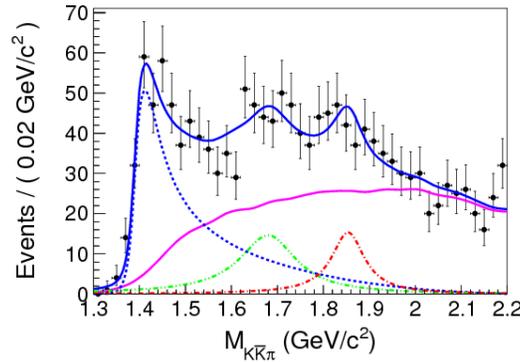


FIGURE 6. The sum of  $K_S^0 K^\pm \pi^\mp$  and  $K^\pm \pi^\mp \pi^0$  mass spectra in the  $\chi_{c1}$  and  $\chi_{c2}$  mass regions. The markers with error bars represent the data; the dash curve the  $h_1(1380)$  signal; the dash-dot-dot curve the  $\phi(1680)$  signal; and the dash-dot curve the  $\phi(1850)$  signal.

## SUMMARY

Based on  $2.92 \text{ fb}^{-1}$  data sample taken at  $\sqrt{s} = 3.773 \text{ GeV}$ ,  $1.06 \times 10^8$   $\psi(3686)$  events and  $1.31 \times 10^9$   $J/\psi$  events collected with BESIII detector at the BEPCII collider, studies have been performed to search for the radiative transitions of  $\psi(3770)$  and rare decays of charmonium, and study light hadron structures and properties. The upper limits on branching fractions are set for radiative transitions of  $\psi(3770)$ ,  $\psi(3770) \rightarrow \gamma\eta_c(\eta_c(2S))$  and  $\psi(3770) \rightarrow \gamma\chi_{c2}$ , the isospin-violating decay  $\chi_{c0,2} \rightarrow \pi^0\eta_c$ , and  $C$ -violation decay  $J/\psi \rightarrow \gamma\gamma, \gamma\phi$ . The OZI-suppressed decay  $J/\psi \rightarrow \pi^0\phi$  is observed for the first time. The isospin-violating decay  $J/\psi \rightarrow \phi\pi^0 f_0(980)$ ,  $f_0(980) \rightarrow \pi^+\pi^-$  is observed with the width of the  $f_0(980)$  obtained from the dipion mass spectrum found to be much smaller than the world average value. The decay  $\chi_{c1,2} \rightarrow \eta' K^+ K^-$  is studied for the first time. Intermediate process  $\chi_{c1} \rightarrow \eta' f_0(980)$ ,  $\chi_{c1} \rightarrow \eta' f_0(1710)$ ,  $\chi_{c2} \rightarrow \eta' f_2'(1525)$  and  $\chi_{c1} \rightarrow K_0^*(1430)^\pm K^\mp$  are observed and their branching fractions are measured. The branching fractions of decay  $\chi_{cJ}$  decaying into  $\phi K^*(892)\bar{K}$  are measured for the first time and the first evidence of the  $h_1(1380)$  in its decay to  $K^*(892)\bar{K}$  is obtained. These measurements provide important information to understand the nature of  $\psi(3770)$ , the isospin-violation mechanism and properties of light hadrons, such as  $f_0(980)$ ,  $K_0^*(1430)$  and  $h_1(1380)$ .

## REFERENCES

- [1] J. L. Rosner, Phys. Rev. D **64**, p. 094002 (2001).
- [2] P. A. Rapidis *et al.*, Phys. Rev. Lett. **39**, p. 526 (1977).
- [3] W. Bacino *et al.*, Phys. Rev. Lett. **40**, p. 671 (1978).
- [4] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. D **76**, p. 122002 (2007).
- [5] M. Ablikim *et al.* (BES Collaboration), Phys. Lett. B **659**, p. 74 (2008).
- [6] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. Lett. **97**, p. 121801 (2006).
- [7] M. Ablikim *et al.* (BES Collaboration), Phys. Lett. B **641**, p. 145 (2006).
- [8] D. Besson *et al.* (CLEO Collaboration), Phys. Rev. Lett. **104**, p. 159901(E) (2010).
- [9] J. Z. Bai *et al.* (BES Collaboration), Phys. Lett. B **605**, p. 63 (2005).
- [10] N. E. Adam *et al.* (CLEO Collaboration), Phys. Rev. Lett. **96**, p. 082004 (2006).
- [11] T. E. Coan *et al.* (CLEO Collaboration), Phys. Rev. Lett. **96**, p. 182002 (2006).
- [12] R. A. Briere *et al.* (CLEO Collaboration), Phys. Rev. D **74**, p. 031106 (2006).
- [13] G. S. Adams *et al.* (CLEO Collaboration), Phys. Rev. D **73**, p. 012002 (2006).
- [14] K. A. Olive *et al.* (Particle Data Group), Chin. Phys. C **38**, p. 090001 (2014).
- [15] G. Li and Q. Zhao, Phys. Rev. D **84**, p. 074005 (2011).
- [16] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **89**, p. 112005 (2014).
- [17] R. E. J. J. Dudek and C. E. Thomas, Phys. Rev. D **79**, p. 094504 (2009).
- [18] D. H. Q. Y. B. Ding and K. T. Chao, Phys. Rev. D **44**, p. 3562 (1991).
- [19] J. L. Rosner, Phys. Rev. D **64**, p. 094002 (2001).
- [20] K. L. E. J. Eichten and C. Quigg, Phys. Rev. D **69**, p. 094019 (2004).
- [21] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **91**, p. 092009 (2015).
- [22] G. L. U. G. M. F. K. Guo, C. Hanhart and Q. Zhao, Phys. Rev. D **82**, p. 034025 (2010).
- [23] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **91**, p. 112018 (2015).
- [24] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **90**, p. 092002 (2014).
- [25] G. S. Adams *et al.* (CLEO Collaboration), Phys. Rev. Lett. **101**, p. 101801 (2008).
- [26] H. E. Haber and J. Perrier, Phys. Rev. D **32**, p. 2961 (1985).
- [27] H. F. W. S. A. Seiden and H. E. Haber, Phys. Rev. D **38**, p. 824 (1988).
- [28] R. Escribano *et al.* (CLEO Collaboration), Eur. Phys. J. C **65**, p. 467 (2010).
- [29] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **91**, p. 112001 (2015).
- [30] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **108**, p. 182001 (2012).
- [31] J. J. Wu, X. H. Liu, Q. Zhao, and B. S. Zou, Phys. Rev. Lett. **108**, p. 081803 (2012).
- [32] F. Aceti, W. H. Liang, E. Oset, J. J. Wu, and B. S. Zou, Phys. Rev. D **86**, p. 114007 (2012).
- [33] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **92**, p. 012007 (2015).
- [34] F. Aceti, J. M. Dias, and E. Oset, Eur. Phys. J. A **51**, p. 48 (2015).
- [35] G. Bonvicini *et al.* (CLEO Collaboration), Phys. Rev. D **78**, p. 052001 (2008).
- [36] D. V. Bugg, Phys. Lett. B **632**, p. 471 (2006).
- [37] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **89**, p. 074030 (2014).
- [38] B. M. D. M. Li and H. Yu, Eur. Phys. J. direct A **26**, p. 141 (2005).
- [39] S. Godfrey and N. Isgur, Phys. Rev. D **32**, p. 189 (1985).
- [40] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **91**, p. 112008 (2015).