Cosmological Scaling Solutions and Cross-coupling Exponential Potential

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Abstract

We present a phase-space analysis of cosmology containing multiple scalar fields with a positive or negative cross-coupling exponential potential. We show that there exist power-law kinetic-potential-scaling solutions for a sufficiently flat positive potential or for a steep negative potential. The former is the unique late-time attractor, but it is difficult to yield assisted inflation. The later is never stable in an expanding universe. Moreover, for a steep negative potential there exists a kinetic-dominated regime in which each solution is a late-time attractor. In the presence of ordinary matter these scaling solutions with a negative cross-coupling potential are found unstable. We briefly discuss the physical consequences of these results.

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1 Introduction

Scalar field cosmological models are of great importance in modern cosmology. The dark energy is attributed to the dynamics of a scalar field, which convincingly realizes the goal of explaining current accelerating expansion of universe generically using only attractor solutions [1]. Furthermore a scalar field can drive an accelerated expansion and thus provides possible models for cosmological inflation in the early universe [2]. In particular, there have been a number of studies of spatially homogeneous scalar field cosmological models with an exponential potential. They are already known to have interesting properties; for example, if one has a universe containing a perfect fluid and such a scalar field, then for a wide range of parameters the scalar field mimics the perfect fluid, adopting its equation of state [3]. These scaling solutions are attractors at late times [4]. The inflation models and other cosmological consequences of multiple scalar fields have also been considered [5, 6].

The scale-invariant form makes the exponential potential particularly simple to study analytically. There are well-known exact solutions corresponding to power-law solutions for the cosmological scale factor $a \propto t^p$ in a spatially flat Friedmann-Robertson-Walker (FRW) model [7]. More generally the coupled Einstein-Klein-Gordon equations for a single field can be reduced to a one-dimensional system which makes it particularly suitable for a qualitative analysis [8, 9]. Recently, adopting a system of dimensionless dynamical variables [10], the cosmological scaling solutions with positive or negative exponentials have been studied [11]. In general there are many scalar fields with exponential potentials in supergravity, superstring and the generalized Einstein theories, thus multiple potentials may be more interesting. In the previous paper [12], We studied the stability of cosmological scaling solutions in an expanding universe model with multiple scalar fields with positive or negative exponential potentials. A phase-space analysis of the spatially flat FRW models shows that there exist cosmological scaling solutions which are the unique late-time attractors and successful inflationary solutions driven by multiple scalar fields with a wide range of each potential slope parameter . It is assumed that there is no direct coupling between potentials. Multiple cross-coupling exponential potentials arise in many occasions, for instance, from compactifications of vacuum Einstein gravity on product spaces [13]. Indeed they are a natural outcome of the compactification of higher dimensional theories down to 3+1 dimensions. With this in mind it is worth investigating such potential in a bit more detail.

In this paper, we first study a system of dimensionless dynamical variables of multiple scalar fields with a positive or negative cross-coupling exponential potential. We obtain the scaling solutions and analyze their stability. There still exist cosmological scaling solutions which are the unique late-time attractors. In this model we then introduce a barotropic fluid to the system. We discuss the physical consequences of these results.

2 Cross-coupling Exponential Potential

We consider n scalar fields ϕ_i with a cross-coupling potential

$$V = V_0 \exp\left(-\sum_{i=1}^{n} \lambda_i \kappa \phi_i \right); \tag{1}$$

where $\kappa^2 \equiv 8\pi G_N$ is the gravitational coupling and λ_i are dimensionless constants characterising the slope of the potential. Further we assume all $\lambda_i \geq 0$ since we can make them positive through $\phi_i \to -\phi_i$ if some of them are negative. The evolution equation of each scalar field for a spatially flat FRW model with Hubble parameter H is

$$\ddot{\phi}_i + 3H\dot{\phi}_i - \lambda_i \kappa V = 0; \tag{2}$$

subject to the Friedmann constraint

$$H^2 = \frac{\kappa^2}{3}\left(\sum_{i=1}^{n} \frac{1}{2}\dot{\phi}_i^2 + V \right): \tag{3}$$

Defining $(n+1)$ dimensionless variables

$$x_i = \frac{\kappa \dot{\phi}_i}{\sqrt{6} H}; \qquad y = \frac{\kappa \sqrt{|V|}}{\sqrt{3} H}; \tag{4}$$

the evolution equations (2) can be written as an autonomous system:

$$x_i' = -3x_i\left(1 - \sum_{j=1}^{n} x_j^2\right) \pm \lambda_i \sqrt{\frac{3}{2}} y^2; \tag{5}$$

$$y' = y\left(\sum_{j=1}^{n} 3x_j^2 - \lambda_j \sqrt{\frac{3}{2}} x_j \right); \tag{6}$$

where a prime denotes a derivative with respect to the logarithm of the scalar factor, $N \equiv \ln a$, and the constraint equation (3) becomes

$$\sum_{i=1}^{n} x_i^2 \pm y^2 = 1: \tag{7}$$

Throughout this paper we will use upper/lower signs to denote the two distinct cases of $\pm V_0 > 0$. x_i^2 measures the contribution to the expansion due to the field's kinetic energy density, while $\pm y^2$ represents the contribution of the potential energy. We will restrict our discussion of the existence and stability of critical points to expanding universes with $H > 0$, i.e., $y \geq 0$. Critical points correspond to fixed points where $x_i' = 0$ and $y' = 0$, and there are self-similar solutions with

$$\frac{\dot{H}}{H^2} = -3\sum_{i=1}^{n} x_i^2: \tag{8}$$

This corresponds to an expanding universe with a scale factor $a(t)$ given by $a \propto t^p$, where

$$p = \frac{1}{3\sum_{i=1}^{n} x_i^2} : \tag{9}$$

The system (5) and (6) has at most one n-dimensional sphere S embedded in $(n+1)$-dimensional phase-space corresponding to kinetic-dominated solutions, and a fixed point A, which is a kinetic-potential-scaling solution listed in Table 1.

In order to study the stability of the critical points, using the Friedmann constraint equation (7) we first reduce Eqs.(5) and (6) to n independent equations

$$x_i' = \left(\lambda_i \sqrt{\frac{3}{2}} - 3x_i\right)\left(1 - \sum_{j=1}^{n} x_j^2\right) : \tag{10}$$

Substituting linear perturbations $x_i \to x_i + \delta x_i$ about the critical points into Eqs.(10), to first-order in the perturbations, gives equations of motion

$$\delta x_i' = -2\left(\lambda_i \sqrt{\frac{3}{2}} - 3x_i\right)\sum_{j=1}^{n}(x_j\,\delta x_j) - 3\left(1 - \sum_{j=1}^{n} x_j^2\right)\delta x_i ; \tag{11}$$

which yield n eigenvalues m_i. Stability requires the real part of all eigenvalues being negative.

S: $\sum_{i=1}^{n} x_i^2 = 1$, $y = 0$. These kinetic-dominated solutions always exist for any form of the potential, which are equivalent to stiff-fluid dominated evolution with $a \propto t^{1/3}$ irrespective of the nature of the potential. Then Eqs.(11) become

$$\delta x_i' = -2\left(\lambda_i \sqrt{\frac{3}{2}} - 3x_i\right)\sum_{j=1}^{n}(x_j\,\delta x_j);$$

which yield n eigenvalues: one of them, say m_1, does not vanish, $m_1 = \sqrt{6}\left(\sum_{i=1}^{n}\lambda_i x_i - \sqrt{6}\right)$; the remains of them vanish. Thus the solutions are marginally stable for $\sum_{i=1}^{n}(\lambda_i x_i) > \sqrt{6}$. For the special case $\lambda_i = \lambda$, using the constraint equation (7) we find $\sqrt{6}/(n\lambda) < \sum_{i=1}^{n} x_i/n \leq \left(\sum_{i=1}^{n} x_i^2/n\right)^{1/2} = 1/\sqrt{n}$. That is, if each scalar field has an identical-slope potential, there exist stable points only for $\lambda^2 > 6/n$.

A: $x_i = \frac{\lambda_i}{\sqrt{6}}$, $y = \sqrt{\left(1 - \frac{1}{6}\sum_{i=1}^{n}\lambda_i^2\right)}$. The potential-kinetic-scaling solution exists for sufficiently flat $\sum_{i=1}^{n}\lambda_i^2 < 6$ positive potentials or steep $\sum_{i=1}^{n}\lambda_i^2 > 6$ negative potentials. The power-law exponent, $p = \frac{2}{\sum_{i=1}^{n}\lambda_i^2}$, depends on parameter λ_i. From Eqs.(11) we find the eigenvalues

$$m_i = -\frac{1}{2}\left(6 - \sum_{j=1}^{n}\lambda_j^2\right) :$$

Label	x_i	y	Existence	Stability
S	$\sum_{i=1}^{n} x_i^2 = 1$	0	all λ_i	stable $\sum_{i=1}^{n} (\lambda_i x_i) > \sqrt{6}$
A_+	$\frac{\lambda_i}{\sqrt{6}}$	$\sqrt{(1 - \sum_{i=1}^{n} \frac{\lambda_i^2}{6})}$	$\sum_{i=1}^{n} \lambda_i^2 < 6, V > 0$	stable
A_-	$\frac{\lambda_i}{\sqrt{6}}$	$\sqrt{(\sum_{i=1}^{n} \frac{\lambda_i^2}{6} - 1)}$	$\sum_{i=1}^{n} \lambda_i^2 > 6, V < 0$	unstable

Table 1: The properties of the critical points in a spatially flat FRW universe containing n scalar fields with the cross-coupling exponential potential.

Thus the scaling solution is always stable when this point exists for a positive potential, which corresponds to the power-law inflation in an expanding universe when $\sum_{i=1}^{n} \lambda_i^2 < 2$. However, this solution is unstable for a negative potential.

The different regions of λ_i parameter space lead to different qualitative evolution. As an example we consider the cosmologies containing n scalar fields with the cross-coupling potential $\lambda_i = \lambda$. For the sufficiently flat ($\lambda^2 < 6/n$) positive potential, these kinetic-dominated solutions are unstable and the kinetic-potential-scaling solution is the stable late-time attractor. Hence generic solutions start in the former and approach the later at late times. For the steep ($\lambda^2 > 6/n$) positive potential, there exists a stable kinetic-dominated regime, in which each points are the late-time attractors. Hence generic solutions start in kinetic-dominated solution and approach the stable regime. For the flat sufficiently ($\lambda^2 < 6/n$) negative potential, only these kinetic-dominated solutions exist which are unstable scaling solutions. For the steep ($\lambda^2 > 6/n$) negative potential, the kinetic-potential-scaling solution is unstable and there exists a stable kinetic-dominated regime. Hence generic solutions start in a kinetic-dominated regime or the kinetic-potential-scaling solution and approach the stable kinetic-dominated regime at late times.

3 Plus a Barotropic Fluid

We now consider multiple scalar fields with the cross-coupling potential (1) evolving in a spatially flat FRW universe containing a fluid with barotropic equation of state $P_\gamma = (\gamma - 1)\rho_\gamma$, where γ is a constant, $0 < \gamma \le 2$, such as radiation ($\gamma = 4/3$) or dust ($\gamma = 1$). The evolution equation for the barotropic fluid is

$$\dot{\rho}_\gamma = -3H(\rho_\gamma + P_\gamma); \qquad (12)$$

subject to the Fridemann constraint

$$H^2 = \frac{\kappa^2}{3}\left(\sum_{i=1}^{n} \frac{1}{2}\dot{\phi}_i^2 + V + \rho_\gamma\right): \qquad (13)$$

We define another dimensionless variable $z \equiv \frac{\sqrt{\rho_\gamma}}{\sqrt{3}H}$. The evolution equations (2) and (12) can then be written as an autonomous system:

$$x_i' = -3x_i\left(1-\sum_{j=1}^{n} x_j^2-\frac{\gamma}{2}z^2\right)-\lambda_i\sqrt{\frac{3}{2}}y^2; \tag{14}$$

$$y' = y\left(3\sum_{i=1}^{n} x_i^2+\frac{3}{2}\gamma z^2-\sqrt{\frac{3}{2}}\sum_{i=1}^{n}\lambda_i x_i\right); \tag{15}$$

$$z' = \frac{3}{2}z\left(-\gamma+2\sum_{i=1}^{n} x_i^2+\gamma z^2\right); \tag{16}$$

and the constraint equation becomes

$$\sum_{i=1}^{n} x_i^2 + y^2 + z^2 = 1. \tag{17}$$

Critical points correspond to fixed points where $x_i' = 0$, $y' = 0$ and $z' = 0$, and there are self-similar solutions with

$$\frac{\dot{H}}{H^2} = -3\sum_{i=1}^{n} x_i^2 - \frac{3}{2}\gamma z^2. \tag{18}$$

This corresponds to an expanding universe with a scale factor $a(t)$ given by $a \propto t^p$, where

$$p = \frac{2}{6\sum_{i=1}^{n} x_i^2 + 3\gamma z^2}. \tag{19}$$

The system (14)–(16) has at most one n-dimensional sphere S embedded in $(n+2)$-dimensional phase-space corresponding to kinetic-dominated solutions, a fixed point A which is a kinetic-potential-scaling solution, a fixed point B which is a fluid-dominated solution, and a fixed point C which is a fluid-potential-kinetic-scaling solution listed in Table 2.

S: $\sum_{i=1}^{n} x_i^2 = 1$, $y = 0$, $z = 0$. These kinetic-dominated solutions always exist for any form of the potential, which are equivalent to stiff-fluid dominated evolution with $a \propto t^{1/3}$ irrespective of the nature of the potential. The linearization of system (14)–(16) about these fixed points yields

$$\delta x_i' = 2\left(-\lambda_i\sqrt{\frac{3}{2}} - 3x_i\right)\sum_{j=1}^{n}(x_j\,\delta x_j);$$

$$\delta z' = \frac{3}{2}(2-\gamma)\,\delta z;$$

which indicate that the solutions are marginally stable for $\sum_{i=1}^{n}(\lambda_i x_i) > \sqrt{6}$ and a stiff fluid $\gamma = 2$.

A : $x_i = \frac{\lambda_i}{\sqrt{6}}$, $y = \sqrt{\left(1 - \frac{1}{6}\sum_{i=1}^n \lambda_i^2\right)}$, $z = 0$. The potential-kinetic-scaling solution exists for sufficiently flat $\sum_{i=1}^n \lambda_i^2 < 6$ positive potentials or steep $\sum_{i=1}^n \lambda_i^2 > 6$ negative potentials. The power-law exponent, $p = \frac{2}{\sum_{i=1}^n \lambda_i^2}$, depends on the slope of the potential. The linearization of system (14)–(16) about this critical point yields $(n+1)$ eigenvalues

$$m_i = \frac{1}{2}\left(-6 + \sum_{j=1}^n \lambda_j^2\right);$$

$$m_z = \frac{1}{2}\left(-3\gamma + \sum_{j=1}^n \lambda_j^2\right):$$

Thus the scaling solution is stable for a positive potential with $\sum_{i=1}^n \lambda_i^2 < 3\gamma$, which corresponds to the power-law inflation in an expanding universe when $\sum_{i=1}^n \lambda_i^2 < 2$.

B : $x_i = 0$, $y = 0$, $z = 1$. The fluid-dominated solution exists for any form of the potential, corresponding to a power-law solution with $p = 2/3\gamma$.

$$x_i' = -3\, \delta x_i + \left(3\gamma - \sqrt{6}\lambda_i\right) \delta z;$$

$$z' = -3\gamma\, \delta z;$$

which indicate that the solution is never stable.

C : $x_i = \frac{\sqrt{\frac{3}{2}}\gamma\lambda_i}{\sum_{j=1}^n \lambda_j^2}$, $y = \sqrt{\frac{\frac{3}{2}\gamma(2-\gamma)}{\sum_{i=1}^n \lambda_i^2}}$, $z = \sqrt{1 - \frac{3\gamma}{\sum_{i=1}^n \lambda_i^2}}$. The fluid-potential-kinetic-scaling solution exists for a positive potential with $\sum_{i=1}^n \lambda_i^2 > 3\gamma$. The power-law exponent, $p = 2/3\gamma$, is identical to that of the fluid-dominated solution, depends only on the barotropic index γ and is independent of the slope λ_i of the potential. The linearization of system (14)–(16) about the fixed point yields

$$x_i' = 3(2-\gamma)x_i \sum_{j=1}^n (x_j\, \delta x_j) - 3\left(\left(1 - \frac{\gamma}{2}\right)\left(1 - \sum_{j=1}^n x_j^2\right) + \frac{\gamma}{2}y^2\right)\delta x_i + \left(\sqrt{6}\lambda_i - 3\gamma x_i\right) y\, \delta y;$$

$$y' = 3(2-\gamma) y \sum_{j=1}^n (x_j\, \delta x_j) - \sqrt{\frac{3}{2}}\, y \sum_{j=1}^n (\lambda_j\, \delta x_j) + \left(\frac{3}{2}(2-\gamma)\sum_{j=1}^n x_j^2 - \sqrt{\frac{3}{2}}\sum_{j=1}^n (\lambda_j x_j) + \frac{3}{2}\gamma - \frac{9}{2}\gamma y^2\right)\delta y;$$

which yield $(n+1)$ eigenvalues

$$m_1 = \frac{3(2-\gamma)}{4}\left(-1 + \sqrt{1 - \frac{8\gamma\left(\sum_{i=1}^n \lambda_i^2 - 3\gamma\right)}{\sum_{i=1}^n \lambda_i^2 (2-\gamma)}}\right);$$

$$m_2 = \frac{3(2-\gamma)}{4}\left(-1 - \sqrt{1 - \frac{8\gamma\left(\sum_{i=1}^n \lambda_i^2 - 3\gamma\right)}{\sum_{i=1}^n \lambda_i^2 (2-\gamma)}}\right);$$

Label	x_i	y	z	Existence	Stability
S	$\sum_i^n x_i^2 = 1$	0	0	all λ_i;	stable $\gamma = 2$, $\sum_i^n(\lambda_i x_i) > \sqrt{6}$
A_+	$\frac{\lambda_i}{\sqrt{6}}$	$\sqrt{(1 - \sum_i^n \frac{\lambda_i^2}{6})}$	0	$\sum_i^n \lambda_i^2 < 6$	stable ($\sum_i^n \lambda_i^2 < 3\gamma$)
A_-	$\frac{\lambda_i}{\sqrt{6}}$	$\sqrt{(\sum_i^n \frac{\lambda_i^2}{6} - 1)}$	0	$\sum_i^n \lambda_i^2 > 6$	unstable ($V < 0$)
B	0	0	1	all λ_i;	unstable
C	$\sqrt{\frac{3}{2}}\frac{\gamma\lambda_i}{\sum_j^n \lambda_j^2}$	$\sqrt{\frac{3(2-\gamma)\gamma}{2\sum_i^n \lambda_i^2}}$	$\sqrt{1 - \frac{3\gamma}{\sum_i^n \lambda_i^2}}$	$\sum_i^n \lambda_i^2 > 3\gamma$	stable ($V > 0$)

Table 2: The properties of the critical points in a spatially flat FRW universe containing n scalar fields with the cross-coupling exponential potential plus a barotropic fluid.

$$m_3 = \cdots = m_{n+1} = \frac{3}{2}(2 - \gamma):$$

Thus the scaling solution is stable for a positive potential with $\sum_{i=1}^n \lambda_i^2 > 3\gamma$.

The different regions in the $(\gamma;\lambda_i)$ parameter space lead to different qualitative evolution. For the sufficiently flat ($\sum_{i=1}^n \lambda_i^2 < 3\gamma$) positive potential, S, A_+ and B exist. Point A_+ is the stable late-time attractor. Hence generic solutions begin in a kinetic-dominated regime or at the fluid-dominated solution and approach the kinetic-potential-scaling solution at late times. For the intermediate ($3\gamma < \sum_{i=1}^n \lambda_i^2 < 6$) positive potential, all critical points exist. Point C is the stable late-time attractor. Hence generic solutions start in a kinetic-dominated regime, at the kinetic-potential-scaling solution or at the fluid-dominated solution and approach the stable fluid-kinetic-potential-scaling solution. For the steep ($\sum_{i=1}^n \lambda_i^2 > 6$) positive potential, S, B and C exist. Point C is the stable late-time attractor. Hence generic solutions start in a kinetic-dominated regime or at the fluid-dominated solution and approach the stable fluid-kinetic-potential-scaling solution. For the sufficiently flat ($\sum_{i=1}^n \lambda_i^2 < 3\gamma$) negative potential, the kinetic-dominated solution S and the fluid-dominated solution B exist, which are unstable. For the intermediate ($3\gamma < \sum_{i=1}^n \lambda_i^2 < 6$) negative potential, the kinetic-dominated solution S and the fluid-dominated solution B exist, which are unstable. For the steep ($\sum_{i=1}^n \lambda_i^2 > 6$) negative potential, S, A_- and B exist. Point A_- is the stable late-time attractor. Hence generic solutions start in a kinetic-dominated regime or at the fluid-dominated solution and approach the stable kinetic-potential-scaling solution at late times.

4 Conclusions and Discussions

We have presented a phase-space analysis of the evolution for a spatially at FRW universe containing n scalar elds with a positive or negative cross-coupling exponential potential. In particular, for the $_i =$ case, we nd that in the expanding universe model with a su ciently at ($^2 < 6=n$) positive cross-coupling potential the only power-law kinetic-potential-scaling solution is the late-time attractor. It is more di cult to obtain assisted in ation in such models since the elds with cross-coupling exponential potential tend to conspire to act against one another rather than assist each other. However, steep ($^2 > 6=n$) negative cross-coupling potential has kinetic-dominated solutions with $a \propto t^{1=3}$, some of which are the late-time attractors. It can be known that the kinetic energy of each eld tends to be equal via their e ect on the expansion at late times.

Then we have extended the phase-space analysis of the evolution to a realistic universe model with a barotropic uid plus n scalar elds with a positive or negative cross-coupling exponential potential. We have shown that for the su ciently at ($\sum_{i=1}^{n} {}^2_i < 3$) positive cross-coupling potential, the kinetic-potential-scaling solution is the stable late-time attractor. The energy density of the scalar elds dominates at late times. Moreover, for the steep ($\sum_{i=1}^{n} {}^2_i > 6$) positive cross-coupling potential, the uid-kinetic-potential-scaling solution is the stable late-time attractor. However, a negative cross-coupling potential has no stable scaling solutions.

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