Spin symmetry in the anti-nucleon spectrum

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We discuss spin and pseudo-spin symmetry in the spectrum of single nucleons and single anti-nucleons in a nucleus. As an example we use relativistic mean field theory to investigate single anti-nucleon spectra. We find a very well developed spin symmetry in single anti-neutron and single anti-proton spectra. The dominant components of the wave functions of the spin doublet are almost identical. This spin symmetry in anti-particle spectra and the pseudo-spin symmetry in particle spectra have the same origin. However it turns out that the spin symmetry in anti-nucleon spectra is much better developed than the pseudo-spin symmetry in normal nuclear single particle spectra.

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Symmetries in single particle spectra of atomic nuclei have been discussed extensively in the literature, as the violation of spin-symmetry by the spin-orbit term and approximate pseudo-spin symmetry in nuclear single particle spectra: atomic nuclei are characterized by a very large spin-orbit splitting, i.e. pairs of single particle states with opposite spin ($j = l \pm \frac{1}{2}$) have very different energies. This fact allowed the understanding of magic numbers in nuclei and forms the basis of nuclear shell structure. More than thirty years ago [1, 2] pseudo-spin quantum numbers have been introduced by $\tilde{l} = l \pm 1$ and $\tilde{j} = j$ for $j = l \pm \frac{1}{2}$ and it has been observed that the splitting between pseudo-spin doublets in nuclear single particle spectra is by an order of magnitude smaller than the normal spin-orbit splitting.

After the observation that relativistic mean field models yield spectra with nearly degenerate pseudo spin-orbit partners [3], Ginocchio showed clearly that the origin of pseudo-spin symmetry in nuclei is given by a relativistic symmetry in the Dirac Hamiltonian ([4, 5] and references given therein). He found that pseudo-spin symmetry becomes exact in the limiting case, where the strong scalar and vector potentials have the same size but opposite sign. However, this condition is never fulfilled exactly in real nuclei, because in this limit the average nuclear potential vanishes and nuclei are no longer bound. It has been found that the quality of pseudo-spin symmetry is related to the competition between the centrifugal barrier and the pseudo-spin orbital potential [6].

In relativistic investigations a Dirac Hamiltonian is used. In its spectrum one finds single particle levels with positive energies as well as those with negative energies. The latter are interpreted as anti-particles under charge conjugation. This has lead to much efforts to explore configurations with anti-particles and their interaction with nuclei. The possibility of producing a new kind of nuclear system by putting one or more anti-baryons inside ordinary nuclei has recently gained renewed interest [7]. For future studies of anti-particles in nuclei it is therefore of great importance to investigate the symmetries of such configurations.

In a relativistic description nuclei are characterized by two strong potentials, an attactive scalar field $-S(\boldsymbol{r})$ and a repulsive vectror field $V(\boldsymbol{r})$ in the Dirac equation which for nucleons (labelled by a subscript "N") reads,

$$[\boldsymbol{\alpha}\cdot\boldsymbol{p} + V_{\rm N}(\boldsymbol{r}) + \beta(M - S_{\rm N}(\boldsymbol{r}))]\,\psi_{\rm N}(\boldsymbol{r}) = \epsilon_{\rm N}\psi_{\rm N}(\boldsymbol{r}), \tag{1}$$

where $V_{\rm N}(\boldsymbol{r}) = V(\boldsymbol{r})$ and $S_{\rm N}(\boldsymbol{r}) = S(\boldsymbol{r})$. For a spherical system, the Dirac spinor $\psi_{\rm N}$ has the form

$$\psi_{\rm N}(\boldsymbol{r}, s) = \frac{1}{r}\begin{pmatrix} iG_{n\kappa}(r)Y^{l}_{jm}(\theta,\phi,s) \\ -F_{\tilde{n}\kappa}(r)Y^{\tilde{l}}_{jm}(\theta,\phi,s)\end{pmatrix}, \quad j = l \pm \frac{1}{2}, \tag{2}$$

where $Y^{l}_{jm}(\theta,\phi)$ are the spin spherical harmonics. $G_{n\kappa}(r)/r$ and $F_{\tilde{n}\kappa}(r)/r$ form the radial wave functions for the upper and lower components with n and $\tilde{n}$ radial nodes. $\kappa = \langle 1 + \boldsymbol{\sigma}\cdot\mathbf{l}\rangle = (-1)^{j+l+1/2}(j+1/2)$ characterizes the spin orbit operator and the quantum numbers l and j. $\tilde{l} = l - {\rm sign}(\kappa)$ is the orbital angular momentum of the lower component. It is therefore well accepted, that the pseudo-spin quantum number of a particle state with positive energy are nothing but the quantum numbers of the lower component [4, 5].

Charge conjugation leaves the scalar potential $S_{\rm N}(\boldsymbol{r})$ invariant while it changes the sign of the vector potential $V_{\rm N}(\boldsymbol{r})$. That is, for anti-nucleons (labelled by "A"), $V_{\rm A}(\boldsymbol{r}) = -V_{\rm N}(\boldsymbol{r}) = -V(\boldsymbol{r})$ and $S_{\rm A}(\boldsymbol{r}) = S_{\rm N}(\boldsymbol{r}) = S(\boldsymbol{r})$. Charge conjugation of Eq. (2) gives the Dirac spinor for an anti-nucleon,

$$\psi_{\rm A}(\boldsymbol{r}, s) = \frac{1}{r}\begin{pmatrix} -F_{\tilde{n}\tilde{\kappa}}(r)Y^{\tilde{l}}_{jm}(\theta,\phi,s) \\ iG_{n\tilde{\kappa}}(r)Y^{l}_{jm}(\theta,\phi,s)\end{pmatrix}, \quad j = l \pm \frac{1}{2}, \tag{3}$$

with $\tilde{\kappa} = -\kappa$.

We are only interested in positive energy states of the Dirac equations. Therefore normal quantum numbers follow the upper component which is dominant. A particle state is labeled by $\{nl\kappa m\}$, while its pseudo-quantum numbers are $\{\tilde{n}\tilde{l}\tilde{\kappa}m\}$. Following Ref. [8], $\tilde{n} = n + 1$ for $\kappa > 0$; $\tilde{n} = n$ for $\kappa < 0$. An anti-particle state is labeled by $\{\tilde{n}\tilde{l}\tilde{\kappa}m\}$ and its pseudo-quantum numbers are $\{nl\kappa m\}$. In analogy to Ref. [8], we deduce the relation

$$n = \tilde{n} + 1, \text{ for } \tilde{\kappa} > 0; \; n = \tilde{n}, \text{ for } \tilde{\kappa} < 0. \tag{4}$$

With $\kappa(1-\kappa) = \tilde{l}(\tilde{l}+1)$ and $\kappa(1+\kappa) = l(l+1)$ in mind, one derives Schrödinger-like equations for the upper and the lower components

$$\left[-\frac{1}{2M_+}\left(\frac{d^2}{dr^2} + \frac{1}{2M_+}\frac{dV_+}{dr}\frac{d}{dr} - \frac{l(l+1)}{r^2}\right) - \frac{1}{4M_+^2}\frac{\kappa}{r}\frac{dV_+}{dr} + M - V_-\right] G(r) = \begin{cases} +\epsilon_{\rm N} G(r), \\ -\epsilon_{\rm A} G(r), \end{cases} \tag{5}$$

$$\left[-\frac{1}{2M_-}\left(\frac{d^2}{dr^2} - \frac{1}{2M_-}\frac{dV_-}{dr}\frac{d}{dr} + \frac{\tilde{l}(\tilde{l}+1)}{r^2}\right) + \frac{1}{4M_-^2}\frac{\tilde{\kappa}}{r}\frac{dV_-}{dr} + M - V_+\right] F(r) = \begin{cases} -\epsilon_{\rm N} F(r), \\ +\epsilon_{\rm A} F(r), \end{cases} \tag{6}$$

TABLE I: Relation between symmetry and external fields.

	Particle	Anti particle
$dV_+/dr = 0$	Spin symmetry	Pseudo spin symmetry
$dV_-/dr = 0$	Pseudo spin symmetry	Spin symmetry

where $V_\pm(r) = V(r) \pm S(r)$ and $M_\pm = M_\pm(\epsilon) = M \pm \epsilon \mp V_\pm$ with $\epsilon = +\epsilon_{\rm N}$ or $-\epsilon_{\rm A}$. Both equations are fully equivalent to the exact Dirac equation with the full spectrum of particle and anti-particle states. But they carry different quantum numbers. For particle states the first equation carries spin-quantum numbers and the second carries pseudo-spin quantum numbers, for anti-particle states the opposite is true. In the following discussions we will use either the first or the second equations according to the type of quantum numbers (spin or pseudo-spin) we are interested in.

We give the relation between spin or pseudo-spin symmetry and the external fields in Table I. If $dV_+/dr = 0$, we have exact spin symmetry in the particle spectrum and exact pseudo-spin symmetry in the anti-particle spectrum because states with the same l (but different κ) are degenerate in Eq. (5). l is the orbital angular momentum of particle states and pseudo orbital angular momentum of anti-particle states. When $dV_+/dr \neq 0$, the symmetries are broken. But if dV_+/dr is so small that the spin-orbit term (the term $\sim \kappa$) in Eq. (5) is much smaller than the centrifugal term, there will be approximate symmetries. For nuclei far from stability where the nuclear potential is expected to be more diffuse, the spin-orbit splitting in single nucleon spectra will be also smaller as compared to stable nuclei. This quenching of the spin-orbit splitting could be one of the reasons for the change of magic numbers in exotic nuclei.

Similarly, when $dV_-/dr = 0$ in Eq. (6), there is an exact pseudo-spin symmetry in the particle spectra [5]. On the other hand, if we focus on anti-particle states, we have in this case exact spin symmetry because now $\tilde{l}$ is the orbital angular momentum. If $dV_-/dr \neq 0$ but small, we have approximate pseudo-spin symmetry in particle spectra and approximate spin symmetry in anti-particle spectra. This implies that the spin symmetry in the anti-particle spectrum has the same origin as the pseudo-spin symmetry in particle spectrum as realized in Ref. [5]. However, there is an essential difference in the degree to which the symmetry is broken in both cases: the factor $1/M_-^2 = 1/(M - \epsilon + V_-)^2$ is much smaller for anti nucleon states than that for nucleon states. The bound anti-particle energies $\epsilon_{\rm A}$ are in the region between $M - V_+(0) \lesssim \epsilon_{\rm A} \lesssim M$. For realistic nuclei roughly

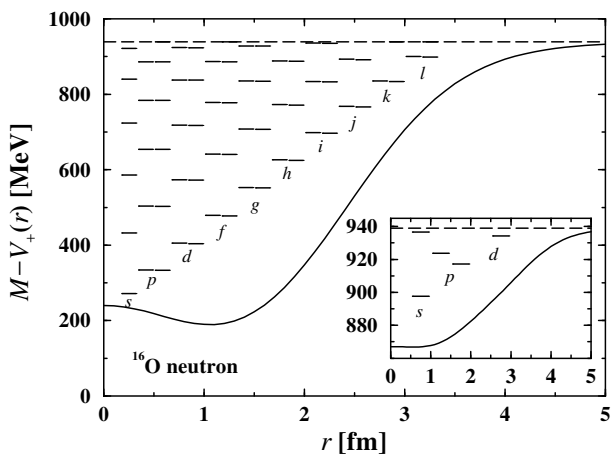

FIG. 1: Anti-neutron potential and spectrum of ^{16}O. For each pair of the spin doublets, the left level is with $\tilde{\kappa} < 0$ and the right one with $\tilde{\kappa} > 0$. The inset gives neutron potential $M + V_-(r)$ and spectrum.

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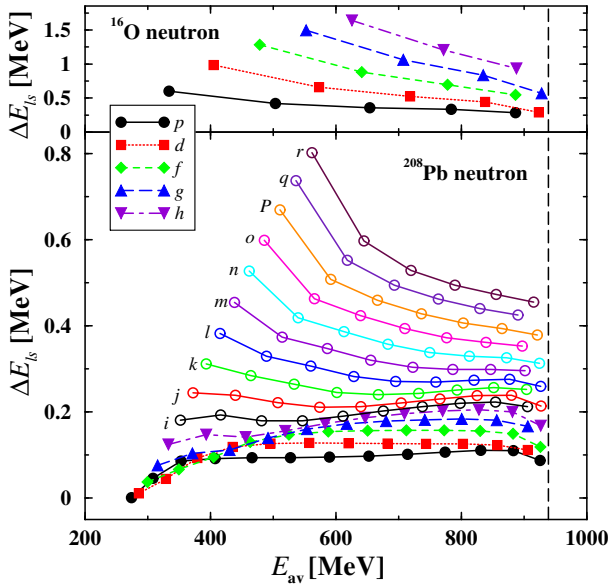

FIG. 2: Spin-orbit splitting $\epsilon_{\rm A}(nl_{l-1/2}) - \epsilon_{\rm A}(nl_{l+1/2})$ in anti-neutron spectra of ^{16}O and ^{208}Pb versus the average energy of a pair of spin doublets. The vertical dashed line shows the continuum limit.

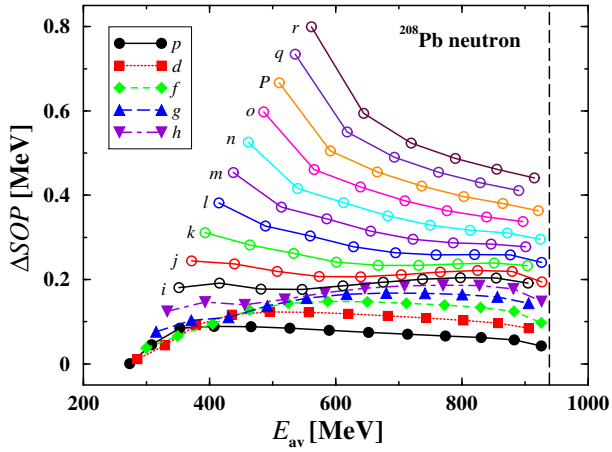

FIG. 3: Difference of the integration of the spin-orbit potential ΔSOP versus the average energy for spin doublets in ^{208}Pb. The vertical dashed line shows the continuum limit.

TABLE II: Energies (in MeV) of some pseudo-spin doublets in neutron spectrum of ^{208}Pb.

$(n+1)s_{1/2}$	$nd_{3/2}$	ΔE	$(n+1)p_{3/2}$	$nf_{5/2}$	ΔE
895.046	898.152	−3.106	904.603	908.520	−3.917
920.168	920.914	−0.746	929.995	930.709	−0.714
938.878	938.455	0.423	$(n+1)f_{7/2}$	$nh_{9/2}$	ΔE
$(n+1)d_{5/2}$	$ng_{7/2}$	ΔE	925.638	927.984	−2.346
914.962	918.517	−3.555	$(n+1)g_{9/2}$	$ni_{11/2}$	ΔE
938.484	938.292	0.192	936.078	936.572	−0.494

we therefore have 0.3 GeV $\lesssim \epsilon_{\rm A} \lesssim$ 1 GeV. On the other hand the bound particle states are in the region of $M - |V_-(0)| \lesssim \epsilon_{\rm N} \lesssim M$, i.e. for realistic nuclei close to 1 GeV. We therefore have $|M_-(\epsilon_{\rm A})| > 2|M - S(0)|$ and $|M_-(\epsilon_{\rm N})| < |V_-(0)|$. Thus the factor in front of the $\tilde{\kappa}$-term is for anti-particle states by more than a factor $(2|M - S(0)|/|V_-(0)|)^2 \approx 400$ smaller than for particle states. Spin-symmetry for anti-particle states is therefore much less broken than pseudo-spin symmetry for particle states.

Since the spin-orbit term in Eq. (6) is so small for anti-nucleon states, we expect in addition that the radial wave functions of the spin-doublets are nearly identical, i.e. the dominant components of spin partners for anti-particle solutions are much more similar than the small components of pseudo-spin partners for particles.

Although the present discussion is meant for single particle spectra in atomic nuclei, the idea is very general. It has been first discovered that the equality of the vector and scalar potentials results in spin symmetry in Ref [9, 10] where the authors suggested applications to meson spectra. However, this symmetry was only recently found to be valid for mesons with one heavy quark [11]. In the present letter, we illustrate for the first time in realistic nuclei nearly exact spin symmetry in the single particle spectra for anti-nucleons. We use for that purpose non-linear relativistic mean field (RMF) theory [12] with modern parameter set NL3. Relativistic Hartree calculations are carried out in coordinate space for the doubly magic nuclei ^{16}O and ^{208}Pb.

For ^{16}O, pseudo-spin symmetry cannot be studied successfully because there are only very few bound nucleon states. However, as seen in Fig. 1, there are many more anti-particle states. We find excellent spin symmetry for them. Since there are too many levels in anti-particle spectra of ^{208}Pb (around 400 for either anti-neutrons or anti-protons), we will not give a similar figure in this case.

In Fig. 2 we present the spin-orbit splitting in anti-neutron spectra of ^{16}O and ^{208}Pb. For ^{16}O, the spin-orbit splittings are around 0.2-0.5 MeV for p states ($l = 1$). With increasing particle number A the spin symmetry in the anti-particle spectra becomes even more exact. For ^{208}Pb, the spin-orbit splittings are ~ 0.1 MeV for p states and less than 0.2 MeV even for h states ($l = 5$) as seen in the lower panel of Fig. 2. We show in Table II the pseudo-spin orbit splitting of the neutron spectrum of ^{208}Pb to compare them with the spin-orbit splitting in anti-nucleon spectra. In most cases, the pseudo-spin orbit splittings for particles are larger than 0.4 MeV and for deeply bound states, it can reach even values around 4 MeV.

In general, the spin-orbit splitting decreases with the state approaching the continuum limit. But for very deeply bound anti-neutron p, d, f and g states in ^{208}Pb, the spin orbit splitting is smaller. This might be due to the competition between the centrifugal barrier and the

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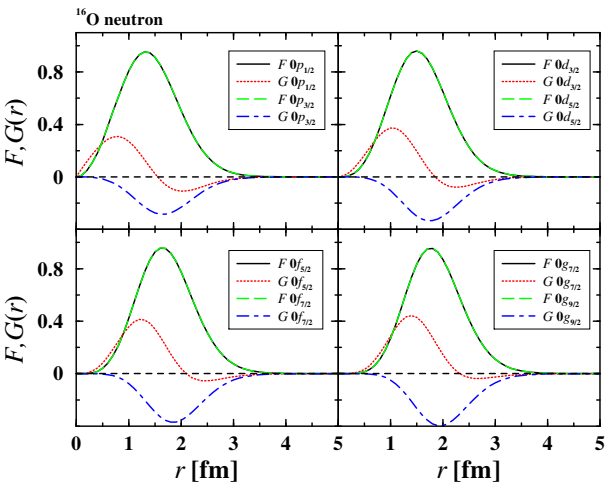

FIG. 4: Radial wave functions of some spin doublets in the anti-neutron spectrum of ^{16}O.

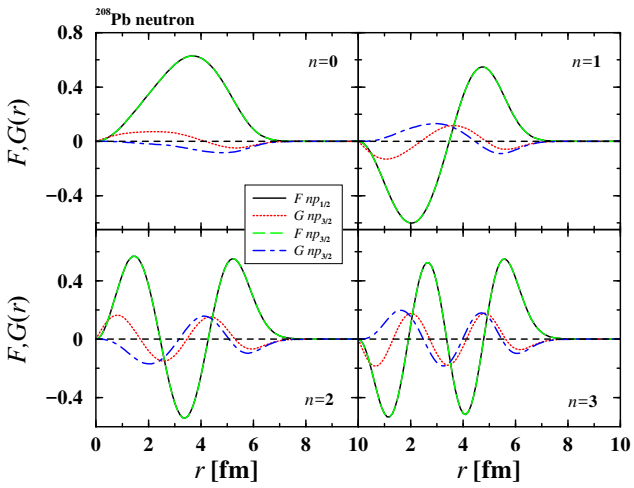

FIG. 5: Radial wave functions of some spin doublets in the anti-neutron spectrum of ^{208}Pb.

spin-orbit potential in Eq. (6). In order to investigate this in more detail, we calculated the expectation value of the spin-orbit potential,

$$SOP = -\int drF(r)^2 \frac{1}{4M_-^2(\epsilon)} \frac{\tilde{\kappa}}{r} \frac{dV_-}{dr}. \qquad (7)$$

Since the lower amplitudes of the two spin doublets are nearly equal to each other (cf. Figs. 4 and 5), we expect the difference, ΔSOP, gives the main part of $\Delta\epsilon$ of a pair of spin doublets. In Fig. 3 we present ΔSOP as a function of the average energy for spin doublets in ^{208}Pb. The variational trend of ΔSOP is roughly in agreement with that of $\Delta\epsilon$. Particularly, for deeply bound states, $\Delta SOP \sim \Delta\epsilon$.

Wave functions of pseudo-spin doublets in single nucleon spectra have been studied extensively in the literature [5]. The lower amplitudes of pseudo-spin doublet are found to be close to each other. Since the spin symmetry in the anti-nucleon spectrum is much more exact than the pseudo-spin symmetry in the single nucleon spectrum, we expect that the upper amplitudes of the spin doublets coincide with each other even much more.

In Figs. 4 and 5, we show radial wave functions $F(r)$ and $G(r)$ for several anti-nucleon spin doublets in ^{16}O and ^{208}Pb. The dominant components $F(r)$ are nearly exactly identical for the two spin partners. On the other hand the small components $G(r)$ of the two spin-partners show dramatic deviations from each other. The relation between the node numbers of the upper and lower amplitudes given in Eq. (4) is seen in Figs. 4 and 5.

In summary, we discussed the relation between the (pseudo)-spin symmetry in single (anti)-particle states and the external fields where the (anti)-particle moves. We present the single anti-nucleon spectra in atomic nuclei as examples and find an almost exact spin symmetry. The origin of the spin symmetry in anti-nucleon spectra and the pseudo-spin symmetry in nucleon spectra have the same origin but the former is much more conserved in real nuclei. We performed RMF calculations for some doubly magic nuclei. Even in a very light nucleus, ^{16}O, the spin symmetry in the anti nucleon spectrum is very good. The spin splitting increases with the orbital quantum number and decreases with the anti-nucleon state approaching the continuum. An investigation of wave functions shows that the dominant components of the Dirac spinor of the anti-nucleon spin doublets are almost identical.

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