Decay width of \(d^*(2380) \rightarrow NN\pi\) process in a chiral constituent quark model

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The width of three-body single-pion decay process \(d^* \rightarrow NN\pi^{0,\pm}\) is calculated by using the \(d^*\) wave function obtained from our chiral SU(3) constituent quark model calculation. The effect of the dynamical structure on the width of \(d^*\) is taken into account in both the single \(\Delta\Delta\) channel and coupled \(\Delta\Delta+CC\) two-channel approximations. Our numerical result shows that in the coupled-channel approximation, namely, the hidden-color configuration being considered, the obtained partial decay width of \(d^* \rightarrow NN\pi\) is about several hundred KeV, while in the single \(\Delta\Delta\) channel it is just about \(2 \sim 3\) MeV. We, therefore, conclude that the partial width in the single-pion decay process of \(d^*\) is much smaller than the widths in its double-pion decay processes. Our prediction may provide a criterion for judging different interpretations of the \(d^*\) structure, as different pictures for the \(d^*\) may result quite different partial decay width.

Keywords: \(d^*(2380)\), Chiral quark model, Strong decay, Single-pion decay

In recent years, CELSIUS/WASA and WASA@COSY Collaborations 1, 2 have reported a clear evidence of a resonance-like structure in double pionic fusion channels \(pn \rightarrow d\pi^0\pi^0\) and \(pn \rightarrow d\pi^+\pi^-\) when they studied the ABC effect 3 and when they treated the neutron-proton scattering data with newly measured analyzing power \(A_y\). Since the observed structure cannot be simply understood by either the intermediate Roper excitation contribution or the t-channel \(\Delta\Delta\) process, they proposed an assumption of existing a \(d^*\) resonance whose quantum number, mass, and width are \(I(J^P) = 0(3^+), M \approx 2370\) MeV and \(\Gamma \approx 70\) MeV 1,2 (see also their recent paper 4, the averaged mass and width are \(M \approx 2375\) MeV and \(\Gamma \approx 75\) MeV, respectively). Due to its baryon number being 2, it would be treated as a dibaryon, and could be explained by either an exotic compact particle or a hadronic molecule 5. Moreover, according to the experimental data, the mass of the \(d^*\) is about 80 MeV below the \(\Delta\Delta\) threshold and about 70 MeV above the \(\Delta\pi N\) threshold, therefore, the threshold (or cusp) effect is expected not to be so significant as that in the XYZ study (see the review of XYZ particles 6). Thus, understanding the internal structure of \(d^*\) would be of great interest.

Actually, the existence of such a non-trivial six-quark configuration with \(I(J^P) = 0(3^+)\) (called \(d^*\) lately) has triggered a great attention and has intensively been studied in the literature since Dyson’s pioneer work 6-12. It should specially be mentioned that one of those calculations reported in 1999 studied the binding behavior of the \(3^+\) dibaryon system by taking into account a \(\Delta\Delta\) channel and a hidden-color channel (denoted by \(CC\) hereafter) simultaneously 11. In that paper, the binding energy was predicted to be about \(40 - 80\) MeV which is quite close to the recent observation, and the importance of the \(CC\) channel was particularly emphasized. Unfortunately, the width of the state was not calculated.

After the discovery of \(d^*\), there are mainly three types of explanation for its nature. Based on the SU(2) quark model, Ref. 13 proposed a \(\Delta\Delta\) resonance structure and performed a multi-channel scattering calculation. They obtained a binding energy of about 71 MeV (namely \(M_{\Delta\Delta} = 2393\) MeV) and a width of about 150 MeV which is apparently much larger than the observation. On the other hand, Ref. 14 studied a three-body system of \(\Delta N\pi\) and found a resonance pole with a mass of \(2363 \pm 20\) MeV and a width of \(65 \pm 17\) MeV. An important view point, claimed by Bashkanov, Brodsky and Clement 15 in 2013, is that a dominant hidden-color structure (or six-quark configuration) of \(d^*\) is necessary for understanding the compact structure of \(d^*\). Sooner after, following our previous prediction 11, Huang and his collaborators made an explicit dynamical calculation by using a chiral SU(3) constituent quark model 16-18 in the framework of the Resonating Group Method (RGM), and showed that the \(d^*\) state has a mass of \(2380 - 2414\) MeV, which agrees with COSY’s observation, and does have an explicit “CC” configuration of about \(66 - 68\%\) in its wave function 12. Based on the obtained wave functions of \(d^*\) and deuteron, Dong and his collaborators calculated the partial decay widths of the “Golden” decay channel \(d^* \rightarrow d + 2\pi^0(\pi^+\pi^-)\) 20 as well as the widths of its four-body decay \(d^* \rightarrow pn\pi^0\pi^0\) and \(d^* \rightarrow pn\pi^+\pi^-\) 21. The results of the two papers showed that inclusion of the \(CC\) configuration inside \(d^*\) would make the calculated widths suppressed greatly and the resultant partial widths for all the \(d^* \rightarrow d\pi^0\pi^0, d^* \rightarrow d\pi^+\pi^-, d^* \rightarrow pn\pi^0\pi^0,\) and \(d^* \rightarrow pn\pi^+\pi^-\) decay processes are well consistent with the experimental data. As a consequence, the total width of \(d^*\) is about \(72\) MeV, which
is fairly close to the observation. All these outcomes support that \( d^* \) is probably a six-quark dominated exotic state due to its large CC component. The general review on the dibaryon studies can be found in Ref. [22] by Clement.

Recently, the questions about how large the decay width of the single pion decay mode of \( d^* \) is and whether such a decay process can be observed have been discussed. Up to now, the \( d^* \to NN\pi \) decay process has not been found in the data analysis [23], but a sizable cross section of this process was predicted by using a \( \Delta N\pi \) model where the rms of \( d^* \) is about 1.5 fm [24]. This contradictory information encourages us to calculate the partial decay width of this process in the same way employed in calculating partial decay widths of the \( d\pi\pi \) and \( N N\pi\pi \) processes and with the same \( d^* \) wave function obtained in our \( \Delta\Delta + CC \) model, with which the resultant rms of \( d^* \) is only about 0.8 fm [10][18], and all the calculated partial decay widths of the \( d\pi\pi \) and \( N N\pi\pi \) processes and the total width of \( d^* \) are consistent with the data quite well [21][22]. Therefore, the obtained partial width might be used to distinguish the models for the structure of \( d^* \).

Similar to our previous work [20][21], we employ the phenomenological effective Hamiltonian for the pseudo-scalar interaction among quark, pion, and quark in the non-relativistic approximation

\[
H_{q\pi q} = \frac{g_{q\pi q}}{(2\pi)^3} 2\sqrt{2} \omega \cdot \vec{k}_\pi \cdot \phi, 
\]

where \( g_{q\pi q} \) is the coupling constant, \( \phi \) stands for the \( \pi \) meson field, \( \omega \) and \( \vec{k}_\pi \) are the energy and three-momentum of the \( \pi \) meson, respectively, and \( \sigma(\tau) \) represents the spin (isospin) operator of a single quark. The wave functions of the nucleon and \( \Delta(1232) \) resonance in the conventional constituent quark model can be found in [20]. The experimental data for the decay width of \( \Delta \to \pi NN \) is about 117 MeV, and the theoretical calculation gives \( \Gamma_{\Delta \to \pi NN} = \frac{1}{3} k_\pi^2 (g_{q\pi q} I_o)^2 E_N / M_\Delta \), where \( E_N = \sqrt{M_N^2 + P_N^2} \) is the energy of the outgoing nucleon, \( k_\pi \approx 0.229 \) GeV is the three-momenta of pion, and \( I_o \) denotes the spatial overlap integral of the internal wave functions of the nucleon and the \( \Delta \) resonance, we can extract the coupling constant \( g_{q\pi q} \) (the details can be found in Ref. [21]). Defining \( G = g_{q\pi q} I_o \), the obtained \( G \) value is about 5.41 GeV$^{-1}$.

As mentioned in Refs. [19][22], our model wave function is obtained by dynamically solving the bound-state RGM equation of the six quark system in the framework of the extended chiral \( SU(3) \) quark model, where the one-gluon-exchange and Goldstone Boson exchange interactions between quarks are explicitly considered. Then, by projecting the obtained wave function onto the inner cluster wave functions of the \( \Delta\Delta \) and \( CC \) channels, the wave function of \( d^* \) can be abbreviated to a form of

\[
\Psi_{d^*} = [ \phi_\Delta(\vec{z}_1, \vec{z}_2) \phi_\Delta(\vec{z}_4, \vec{z}_5) \chi_{\Delta\Delta}(\vec{R}) \zeta_{\Delta\Delta} \phi_{CC}(\vec{z}_1, \vec{z}_2) \phi_{CC}(\vec{z}_4, \vec{z}_5) \chi_{CC}(\vec{R}) \zeta_{CC} ]_{(SU1)} = (30),
\]

where \( \phi_\Delta \) and \( \phi_{CC} \) denote the inner cluster wave functions of \( \Delta \) and \( C \) (color-octet particle) in the coordinate space, \( \chi_{\Delta\Delta} \) and \( \chi_{CC} \) represent the channel wave functions in the \( \Delta\Delta \) and \( CC \) channels (in the single \( \Delta\Delta \) channel case, the \( CC \) component is absent), and \( \zeta_{\Delta\Delta} \) and \( \zeta_{CC} \) stand for the spin-isospin wave functions in the hadronic degrees of freedom in the \( \Delta\Delta \) and \( CC \) channels, respectively. It should be specially mentioned that in such a \( d^* \) wave function, two channel wave functions are orthogonal to each other and contain all the totally anti-symmetrization effects implicitly [19].

In terms of the obtained \( d^* \) wave function eq. (2) (its explicit forms have been plotted in Ref. [20]) and we are able to calculate the width of the three-body single-pion decay \( d^* \to NN\pi \). The partial width reads

\[
\Gamma_{d^* \to NN\pi} = \frac{1}{2!} \int d^3 p_1 d^3 p_2 (2\pi)^3 \delta(\Delta E) \left| M(\vec{p}_1, \vec{p}_2) \right|^2,
\]

where \( \left| M(\vec{p}_1, \vec{p}_2) \right|^2 \) stands for the squared transition matrix element with a sum over the polarizations of the final three body states and an average of the ones of the initial state \( d^* \), the factor of 2! is due to the property of the identical particle in the final \( NN \) system, and \( \delta(\Delta E) \) denotes the energy conservation with \( \Delta E = M_{d^*} - \omega_\pi(p) - E_N(p_1) - E_N(-p_1 - k) \), where \( \omega_\pi(k) \) and \( E_N \) represent the energies of the pion and nucleon, respectively.

The transition matrix contains contributions from 12 Feynman diagrams, where the \( \Delta\Delta \) component in the \( d^* \) wave function is responsible for the decay, the pion-exchange is considered for those sub-leading effects, and the intermediate nucleon state is taken into account only. Among these diagrams, 6 of them where the outgoing pion is emitted from \( \Delta_2 \) are drawn in Fig.1, and they are depicted according to the time-order perturbation theory.
FIG. 1: Six possible ways to emit pion only from the $\Delta \Delta$ component of $d^*$ in the $d^* \to NN\pi$ decay process. The outgoing pion with momenta $\vec{k}$ is emitted from $\Delta_2$. The other six sub-diagrams with pion emitted from $\Delta_1$ are similar, and then are not shown here for reducing the size of the figure.

Computing transition matrix elements for all the diagrams in Fig. 1 is straightforward (refer to the discussions in Ref. [25]). For example, the matrix element for Fig. 1(a) can be written as

$$
\mathcal{M}^{(a)}_{d^* \to NN\pi} = \int \frac{d^3q}{2\omega_{k_E} \sqrt{2\omega_k(2\pi)^3}} \delta^3(p_{N_2} + p_{N_1} + k - p_{\Delta_1} - p_{\Delta_2})
$$

$$
\times \tilde{\Psi}_{d^*}(q) \tilde{\mathcal{M}}_{\pi(k_E)N(p_{\Delta_2}) \to N(p_{\Delta_1})} \mathcal{D}_{af} \tilde{\mathcal{M}}_{\Delta_1 \to \pi(k_E)N(p_1)} \mathcal{D}_{ai} \tilde{\mathcal{M}}_{\Delta_2 \to \pi(k)N(p_2)},
$$

where $\Psi_{d^*}$ represents the $d^*$ wave function in the momentum space which can be obtained by Fourier transforming the $d^*$ wave functions in the coordinate space in both the single $\Delta \Delta$ channel and coupled $\Delta \Delta + CC$ channel approximations (see details in Refs. [20]). $\tilde{\mathcal{M}}_{\pi(k_E)N(p_{\Delta_2}) \to N(p_{\Delta_1})}$, $\tilde{\mathcal{M}}_{\Delta_1 \to \pi(k_E)N(p_1)}$, and $\tilde{\mathcal{M}}_{\Delta_2 \to \pi(k)N(p_2)}$ denote the transitions of $\pi(k_E)N(p_{\Delta_2}) \to N(p_{\Delta_1})$, $\Delta_1 \to \pi(k_E)N(p_1)$, and $\Delta_2 \to \pi(k)N(p_2)$, respectively, and the non-relativistic energy propagators are

$$
\mathcal{D}_{af} = \frac{1}{M_{d^*} - \omega(\vec{k}) - \omega(\vec{k}_E) - E_N(\vec{p}_1) - E_N(\vec{p}_2')},
$$

$$
\mathcal{D}_{ai} = \frac{1}{M_{d^*} - \omega(\vec{q}) - E_{\Delta_1}(\vec{q}) - E_N(\vec{p}_1')},
$$

Then the explicit form of the matrix element (Fig. 1(a)) for the case where the spin of the final two-nucleon is zero ($s_{12} = 0$) can be expressed as

$$
\mathcal{M}^{(a)}_{s_{12}=0}(p_1, k) = -G_{\pi NN}G_{\pi \Delta N}^2 \frac{\pi}{2} \frac{2\sqrt{2}}{2\omega_k} Y_{1,0}(\vec{k}) \int \frac{d^3q}{2\omega_{k_E} \sqrt{2\omega_k(2\pi)^3}} \sum_{m_i} Y_{2m_i}(\vec{k}_E) \times \sqrt{\frac{2}{5}} C_{10,2m_i}.\]

In the above equation, $\vec{k}_E = \vec{q} - \vec{p}_1$ stands for the three-momenta of the exchanged pion, $G_{\pi NN} \approx 4\sqrt{2} G$ and $G_{\pi \Delta N} \approx \frac{3g_{\pi N\pi}}{2MN}$ (with $g_{NN\pi} \approx 13.6$), and $\vec{k}_E = \vec{q} - \vec{p}_1$. Moreover, for the case of $s_{12} = 1$, we have

$$
\mathcal{M}^{(a)}_{s_{12}=1}(p_1, k) = -G_{\pi NN}G_{\pi \Delta N}^2 \frac{\pi}{2} \frac{2\sqrt{2}}{2\omega_k} Y_{1,0}(\vec{k}) \int \frac{d^3q}{2\omega_{k_E} \sqrt{2\omega_k(2\pi)^3}} \sum_{m_i} Y_{2m_i}(\vec{k}_E) \times \left[ \sqrt{\frac{1}{5}} C_{10,1m_{s_{12}}} C_{1m_{s_{12}}2m_i} + \sqrt{\frac{2}{3}} C_{10,2m_{s_{12}}} C_{2m_{s_{12}},2m_i} \right].
$$
In general, the final state interaction (FSI) between two outgoing nucleons should be considered. However, being aware of the fact that the quantum numbers of \( d^\ast \) are \( I(J^P) = 0(3^+) \), the maximal spin of two nucleons is 1, and parity \( P \) and total angular momentum \( J \) of the decaying system should be conserved, either the orbital angular momentum between the outgoing pion and nucleon is at least equal to 3, or the orbital angular momentum between two outgoing nucleons at least equals to 2. In the former case, the decay cross section would be greatly suppressed by the higher partial wave. And in the latter case, the FSI effect could be ignored because in the low energy region, the dominant contribution comes from the S-wave and P-wave, and the contribution from the higher partial wave can be ignored \[24\]. Therefore, in this calculation, we assume the enhancement factor from FSI is close to 1, and consequently would not be considered.

Table 1, The calculated decay width of the \( d^\ast \rightarrow pn\pi^0 \) process and the widths contributed individually from the (a)-, (b)-, (c)-, (d)-, (e)-, and (f)-type diagrams (in units of MeV).

<table>
<thead>
<tr>
<th>Case</th>
<th>Total width</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>sum of (a)-(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>One ch. (( \Delta\Delta ) only)</td>
<td>2.276</td>
<td>0.550</td>
<td>0.306</td>
<td>0.267</td>
<td>0.0963</td>
<td>0.209</td>
<td>0.233</td>
<td>1.661</td>
</tr>
<tr>
<td>Two chs. (( \Delta\Delta+CC ))</td>
<td>0.670</td>
<td>0.154</td>
<td>0.0884</td>
<td>0.0789</td>
<td>0.0279</td>
<td>0.0687</td>
<td>0.0847</td>
<td>0.503</td>
</tr>
</tbody>
</table>

The numerical results for the decay width in the \( d^\ast \rightarrow NN\pi \) process and the widths contributed individually from the \( (a)-, (b)-, (c)-, (d)-, (e)-, \) and \( (f)-\) type diagrams are tabulated in Table 1, respectively. Contributions from the diagrams where the outgoing pion being emitted from \( \Delta_1 \) are also taken into account. The total decay width of the \( d^\ast \rightarrow NN\pi \) process is the sum of the contributions from all types of diagrams coherently. From table 1, one sees that the ratio of the decay width with coherent sum to that with incoherent sum is about 1.37 in the one channel (\( \Delta\Delta \) channel only) case and about 1.33 in the two channel (coupled \( \Delta\Delta+CC \) channels) case, respectively, which shows a sizeable coherent effect. The most important issue from this table is that the decay width of \( d^\ast \to pn\pi^0 \) is smaller than 3 MeV in the one channel case and about 670 KeV in the two channel case. Apparently, the width in the two channel case is much smaller than that in the one channel case. Since in the two channel case, the contribution comes from the \( \Delta\Delta \) component of \( d^\ast \) only, which is about 31.5% of the whole \( d^\ast \) wave function, but in the one channel calculation, the contribution comes from the whole \( \Delta\Delta \) wave function, therefore, this outcome is understandable. The second observation is that although the framework and method in this calculation are the same as that used in the \( d^\ast \) width calculations in the double-pion decay processes before, the obtained decay widths for the single-pion decay mode are remarkably smaller than those given in our previous calculations \[21\] and the experimental data for the double-pion decay mode. This is because that in the single pion decay process, the leading non-vanishing contribution comes from the sub-leading diagrams shown in Fig. 1, where three vertices exist, whereas in the double-pion decay process, the leading non-vanishing contribution comes from the two vertices diagrams shown in Refs. \[24\] \[21\]. Clearly, the obtained very small decay width for the single pion decay mode is consistent with the current experimental status that no \( d^\ast \to NN\pi \) process has been found in the present data set.

Some approximations in calculation should be further discussed. In the sub-leading diagrams shown in Fig. 1, the Goldstone-Boson exchange between two nucleons must be introduced to convert two \( \Delta \)s to two nucleons. From PDG \[27\], one finds that the largest decay mode for \( \Delta(1232)3/2^+ \) is \( N\pi \) with a branching ratio of about 100%. Therefore, in the realistic calculation, considering the pion-exchange only would not miss the major feature and make the result meaningless. Due to the larger mass of nucleon excitations, we do not take them as the intermediate nucleon state. We also ignore the contribution from the diagrams where the intermediate \( \Delta \) state exist, because the quark model calculation tells us that the \( \Delta\Delta\pi \) coupling \( f_{\Delta\Delta\pi} \) is much smaller than the \( NN\pi \) coupling \( f_{NN\pi} \) \[28\]. Again, we would specially emphasize that the contributions from the large CC component in the \( d^\ast \) wave function could be ignored. The reason is the following. In the previous paragraph, we have mentioned the extracted channel wave functions for various channels have already absorbed the effect of the totally anti-symmetrization, and the channel wave functions for the \( \Delta\Delta \) and CC channels are orthogonal to each other. So in our decay calculation, the inter-cluster quark exchange should not be considered anymore. In the \( (SI = 30) \) case, where \( S \) and \( I \) denote the spin and isospin of the system, respectively, converting CC to \( \Delta\Delta \) requires an exchange of a colored object, namely a gluon. The calculation shows that without quark exchange, the matrix elements of the one-gluon-exchange interaction (OGE) between \( \Delta\Delta \) and CC are zero, namely CC cannot be converted to \( \Delta\Delta \). If one would convert CC(\( SI = 30 \)) to \( \Delta\Delta(S \neq 3) \) and CC to \( NN \), because the spin of the system is \( S = 3 \) and the parity is positive, it needs at least D-wave between \( \Delta \) and \( \Delta \) with \( (S \neq 3) \) and between \( N \) and \( N \), respectively. Then, the required tensor force in OGE, which is a higher order term, would make these conversions suppressed greatly. In short, we can ignore the contribution from the CC component in \( d^\ast \) in the \( d^\ast \) decay calculation.
To summarize, we proceed a calculation for the single-pion decay mode of $d^*$ with the help of our wave function obtained in the chiral constituent quark model calculation. It shows in our calculations [19–21] that the CC component has a large fraction of about $2/3$ in the wave function of $d^*$, and the rest part, the $\Delta\Delta$ component, is responsible for its widths in the decays of $d^* \rightarrow d\pi\pi$ and $d^* \rightarrow NN\pi\pi$ as well as $d^* \rightarrow NN\pi$. As a result, the obtained partial widths for the double-pion decay modes of $d^*$ are in good agreement with the experimental measurement. Moreover, the obtained width for the single-pion decay model in this calculation is much smaller than those in the double-pion decay modes. If we assume that the total width of the $d^*$ is about 75 MeV, the predicted branching ratio of the single-pion decay mode $d^* \rightarrow pn\pi^0$ is about 3.0% in the one-channel case and 0.9% in the coupled-channel case. It should be emphasized that the obtained single-pion decay width is much smaller than the widths of the double-pion decay modes. This result agrees with the present experiment status that such a single-pion decay mode has not been found in that data analysis. It is quite different from the width reported by the investigation with the $\Delta N\pi$ assumption for the structure of $d^*$, where the predicted width for the single pion decay mode would be large enough to be observed in the experimental measurement. It is expected that an intensive data analysis of the $d^* \rightarrow pn\pi^0$ channel would judge different explanations for the nature of $d^*$.

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