Significance of Energy Scale for Physics at BESIII*

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Abstract The effects of absolute energy calibration on BESIII physics are discussed in detail, which mainly involve the effects on $\tau$ mass measurement, cross section scan measurement, and generic error determination in other measurements.

Key words absolute energy calibration, $\tau$ mass, cross section scan, error estimation

1 Introduction

High luminosity accelerator, the CESR/CLEO-c, has taken huge data sample in charmonium energy region. Another accelerator, namely, the BEPCII has finished its upgrading and the new detector BESIII \cite{1} will be moved into the collision point soon. The designed peak luminosity of BEPCII is $10^{33}$ cm$^{-2}$s$^{-1}$ (1 nb$^{-1}$s$^{-1}$) at beam energy 1.89 GeV, which is the highest in $\tau$-charm energy region ever planned. The great number of events are to be available in the near future, as planned in Table 1, which is estimated by the method in Ref \cite{2} according to the parameters in PDG2006 \cite{3}, and partly the CLEO-c’s results are referenced.

Besides large data sample, the detector performance is to be improved considerably compared with the BESII, refer to Table 2. On the strength of so large data sample together with the excellent detector performance, the unprecedented precision (1%–2% or better) could be expected for lots of physical analyses. For such an accurate analysis, many meticulous factors and effects have to be considered seriously, including the effects due to the accuracy of beam energy.

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Table 1: Number of events expected for one year of running at BEPCII/BESIII. $E_{cm}$ is the center of mass energy, $\mathcal{L}_{\text{peak}}$ the peak luminosity, $\Delta E_{cm}$ the energy spread, $\sigma^{\text{obs}}$ the observed cross section, and $N_{\text{evt}}$ the number of events. It should be noticed that $\sigma^{\text{obs}}$ depends on the actual running status of accelerator ($\Delta E_{cm}$, etc.).

<table>
<thead>
<tr>
<th>Physics</th>
<th>$E_{cm}$ (GeV)</th>
<th>$\mathcal{L}_{\text{peak}}$ (nb$^{-1}$s$^{-1}$)</th>
<th>$\Delta E_{cm}$ (MeV)</th>
<th>$\sigma^{\text{obs}}$ (nb)</th>
<th>$N_{\text{evt}}$ ($10^6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J/\psi$</td>
<td>3.097</td>
<td>0.6</td>
<td>0.93</td>
<td>3200</td>
<td>9.600</td>
</tr>
<tr>
<td>$\tau$</td>
<td>3.670</td>
<td>1.0</td>
<td>1.30</td>
<td>2.4</td>
<td>12</td>
</tr>
<tr>
<td>$\psi(2S)$</td>
<td>3.686</td>
<td>1.0</td>
<td>1.31</td>
<td>700</td>
<td>3.500</td>
</tr>
<tr>
<td>$D$</td>
<td>3.770</td>
<td>1.0</td>
<td>1.37</td>
<td>$6.4^{[4]}$</td>
<td>32</td>
</tr>
<tr>
<td>$D_s$</td>
<td>4.030</td>
<td>0.6</td>
<td>1.57</td>
<td>$0.17^{[5]}$</td>
<td>0.51</td>
</tr>
<tr>
<td>$\bar{D}_s$</td>
<td>4.140</td>
<td>0.6</td>
<td>1.65</td>
<td>0.68 $^{[5]}$</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table 2: Detector parameters comparison between the BESII and the BESIII for different sub-systems $^{[1]}$: MDC (main drift chamber), EMC (electromagnet calorimeter), TOF (time-of-flight detector), MUC ($\mu$-counter) and magnet.

<table>
<thead>
<tr>
<th>Sub-system</th>
<th>Index</th>
<th>BESII</th>
<th>BESIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDC</td>
<td>$\sigma_{xy}$</td>
<td>250 $\mu$m</td>
<td>130 $\mu$m</td>
</tr>
<tr>
<td></td>
<td>$\delta p/p$</td>
<td>2.4$%$ @ 1 GeV/c</td>
<td>0.5$%$ @ 1 GeV/c</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{dE/dx}$</td>
<td>8.5$%$</td>
<td>6$\sim$7$%$</td>
</tr>
<tr>
<td>EMC</td>
<td>$\delta E/E$</td>
<td>20$%$ @ 1 GeV</td>
<td>2.5$%$ @ 1 GeV</td>
</tr>
<tr>
<td></td>
<td>$\sigma_z$</td>
<td>3 cm @ 1 GeV</td>
<td>6 mm @ 1 GeV</td>
</tr>
<tr>
<td>TOF</td>
<td>$\sigma_T$</td>
<td>180 ps (barrel)</td>
<td>90 ps (barrel)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>350 ps (endcap)</td>
<td>110 ps (endcap)</td>
</tr>
<tr>
<td>MUC layers</td>
<td>3</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Magnet</td>
<td>0.4 Tesla</td>
<td>1.0 Tesla</td>
<td></td>
</tr>
</tbody>
</table>
The beam energy is an important parameter for both accelerator and detector. The uncertainty of beam energy is linearly transformed into the systematic error in \( \tau \) mass measurement\(^6\), and will affect the measurement uncertainties of resonance parameters in cross section scan experiment. Moreover, the accuracy of beam energy also plays an unnegligible role in high precision error analysis of generic physics studies at the BESIII. Therefore, it is necessary to determine the beam energy with high accuracy at the BEPCII and the BESIII.

In this paper, we expound the effects of absolute energy calibration on BESIII in these aspects: the effects on \( \tau \) mass measurement scan measurement, and error determination for exclusive analysis.

### 2 Significance for \( \tau \) mass measurement

The mass of the \( \tau \) lepton \((m_\tau)\) is a fundamental parameter in standard model, many experiments have performed to determine it accurately, and some measurements\([7, 8, 9, 10, 11, 12, 13, 14, 15, 16]\) are displayed in Fig. 1. Usually, the pseudomass and threshold-scan methods are employed to measure the \( \tau \) mass, where the latter is adopted by BES Collaboration to achieve the accurately measured value\([12]\):\(^{ } \)

\[
m_\tau = 1776.96^{+0.18+0.25}_{-0.21-0.17} \text{ MeV} .
\]  

(1)

Now it is of great interest to know what accuracy of \( \tau \) mass we can expect for a large \( \tau \) data sample at the BESIII.

Note that in Eq. (1) the relative statistical \((1.6 \times 10^{-4})\) and systematic \((1.7 \times 10^{-4})\) uncertainties have comparable magnitude, so the following estimation is divided into two parts, one about the statistical and the other about the systematic uncertainties.

For statistic estimation, Monte Carlo simulation is employed to study the optimal data taking strategy for the high precision \(m_\tau\) measurement at BESIII. In Ref.\([6]\) the numbers of integrated luminosity and corresponding fit uncertainty are presented, by virtue of which an empirical formula could be fit out:

\[
\delta m_\tau[\text{keV}] = \frac{708}{L^{0.504}} , \quad \text{or} \quad \nu_{m_\tau} = \frac{3.98 \times 10^{-4}}{L^{0.504}} ,
\]  

(2)

where \(L\) denotes the integrated luminosity (in unit of \(\text{pb}^{-1}\)), \(\delta m_\tau\) indicates the fit uncertainty and \(\nu_{m_\tau}\) the relative one. Based on Eq. (2), the luminosity for certainty accuracy requirement can be readily obtained. For example, if \(\delta m_\tau = 0.1 \text{ MeV}\), then \(L\) should be 49 \(\text{pb}^{-1}\); furthermore, if \(\nu_{m_\tau} = 1 \times 10^{-5}\), then \(L\) is at least to be 1500 \(\text{pb}^{-1}\).

The aforementioned designed peak luminosity at BESIII is around 1 \(\text{nb}^{-1}s^{-1}\), if the average efficiency of luminosity is taken as 50% of the peak value, then two days data taking time is sufficient to reach the statistical uncertainty of less than 0.1 \(\text{MeV}\); and 35
days of data taking time can lead to a relative statistical uncertainty of less than $1 \times 10^{-5}$.

Notice that these estimations are solely for $e\mu$-tagged events, if more channels are utilized to tag $\tau$-pair final state, such as $ee$, $e\mu$, $eh$, $\mu\mu$, $\mu h$ ($h$: hadron), and so on, more statistics can be expected and shorter time will be needed for the actual data taking\(^1\).

Next, we turn to systematic uncertainty. Table 3 summarizes some possible systematic uncertainties\(^{[17]}\) in $m_\tau$ measurement, which include the uncertainties due to different theoretical formulas utilized in the fit, energy spread effects, luminosity, efficiency, $\tau$ decay branching fraction, background and so on. Besides all these uncertainties, another important factor should be taken into account, that is the absolute value of beam energy. As displayed in Fig. 2, the uncertainty of the center of mass energy determined by the uncertainty of the beam energy will be transferred to the final fit result of $m_\tau$ directly and linearly, so the absolute determination of beam energy is actually a bottleneck in accuracy improvement for $m_\tau$ measurement. Moreover, unlike the uncertainties of luminosity and efficiency, which can be improved further with large data sample, the accuracy of beam energy is determined by the hardware measurement and has nothing to do with the size of data sample.

At present, two methods have been used to determine the absolute beam energy accurately, one is the depolarization method and the other is the Compton backscattering

\(^{1}\)According to the previous BES analyses\(^{[12, 18]}\) experience, the number of multi-channel-tagged events is at least 5 times more than that of the $e\mu$-tagged events.
Table 3: Some possible systematic uncertainties in $m_\tau$ measurement.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\delta m_\tau$ ($10^{-3}$ MeV)</th>
<th>$\delta m_\tau/m_\tau$ ($10^{-6}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminosity</td>
<td>14.0</td>
<td>7.9</td>
</tr>
<tr>
<td>Efficiency</td>
<td>14.0</td>
<td>7.9</td>
</tr>
<tr>
<td>Branching fraction</td>
<td>3.5</td>
<td>2.0</td>
</tr>
<tr>
<td>Background</td>
<td>1.7</td>
<td>1.0</td>
</tr>
<tr>
<td>Energy spread</td>
<td>3.0</td>
<td>1.7</td>
</tr>
<tr>
<td>Theoretical accuracy</td>
<td>3.0</td>
<td>1.7</td>
</tr>
<tr>
<td>Energy scale</td>
<td>100</td>
<td>56.3</td>
</tr>
<tr>
<td>Summation</td>
<td>102</td>
<td>57.5</td>
</tr>
</tbody>
</table>

method, both have been utilized by the KEDR group [16]. It proposed to adopt the Compton backscattering technique to measure the energy at the BESIII [19], the relative accuracy of such technique is expected to be at the level of $5 \times 10^{-5}$. If such a system is established at the BEPCII, the systematic uncertainty can be expected at this level, that is $5 \times 10^{-5}$ (relative error) or around 0.09 MeV (absolute error). Anyway, compared with the statistical error or other systematic error listed in Table 3, it is obvious that the final accuracy of $m_\tau$ measurement at the BESIII is mainly determined by the accuracy of beam energy measurement.

3 Significance for cross section scan experiment

One kind of important experiments of high energy physics is the measurement of resonance parameters by a cross section scan in the vicinity of the resonance, which can provide the fundamental information about a resonance, such as mass, total decay width, partial decay width and so on. Moreover, some special physics analyses can only be finished based on the scan experiment, for example, the phase between strong and electromagnetic interaction could only be measured by the scan experiment model-independently [20, 21]. As a matter of fact, the uncertainty of beam energy has effect on all fit parameters determined by scan experiment. Next we take the scan experiment in the vicinity of $\psi(2S)$ as an example, to give a special estimation.

The following $\chi^2$ estimator is often constructed to obtain resonance parameters [22, 23]

$$\chi^2 = \sum_{j=1}^{n_{ch}} \sum_{i=1}^{n_{pt}} \frac{(N_i^j - L_i \cdot e^j \cdot \sigma^j(E_i, \vec{\eta}))^2}{(\Delta N_i^j)^2},$$

(3)

where $N_i^j$ ($\Delta N_i^j$), $L_i$, $e^j$, and $\sigma^j(E_i, \vec{\eta})$ indicate the observed number of events (corre-
Figure 2: The effect due to the uncertainty of absolute energy calibration on $m_\tau$ measurement.

The integrated luminosity, the efficiency, and the observed cross section, respectively, and the cross section depends on the energy and resonance parameters. Here the superscript $j$ indicates different channels while the subscript $i$ different energy point where the data are taken\(^2\). ($n_{pt}$ denotes the total number of points taken for certainty scan experiment). For the observed cross section, it depends on the energy point and other resonance parameters (denoted by $\vec{\eta}$). The energy $E_i$ is the value provided by accelerator measurement system but the actual energy point may be shifted within the energy-measured uncertainty. We estimate such kind of uncertainty with the help of Monte Carlo simulation [22]. Specially, for certain assumed experiment, the energy point $E_i$ is given by the sampling method:

$$E_i = E_i^0 + \xi_i \cdot \delta E,$$

where $E_i^0$ indicates the nominal energy, $\delta E$ the energy uncertainty, and $\xi$ is a random number in a Gaussian distribution with mean zero and variance one [24]. Here we have assumed the energy uncertainty for all scan points is the same. With a set of energy point and minimizing Eq. (3), we could get a group of fitting parameters ($\vec{\eta}$). Similarly, another sampling leads to another set of fitting parameters. So we continue such an process until we acquire the distributions for every resonance parameters. Fitting the distributions, we could get the corresponding error. Figure 3 shows the effect of energy uncertainty on the resonance parameters such as $\Gamma_{tot}$, $\Gamma_{\pi^+\pi^- J/\psi}$ and $\Gamma_{\mu^+\mu^-}$. It could be seen that the effect due to the energy uncertainty increases almost linearly.

\(^2\)The dependence of $\sigma$ on energy is denoted by $E_i$, and on resonance parameters by $\vec{\eta}$ which includes the total decay width $\Gamma_{tot}$, partial decay width for $\pi^+\pi^- J/\psi$ $\Gamma_{\pi^+\pi^- J/\psi}$, partial decay width for $\mu^+\mu^-$ final state $\Gamma_{\mu^+\mu^-}$, and so forth.
Table 4 list the various errors for resonance parameters, by virtue of which we could see at the forthcoming new experiment that the effect due to the accuracy of energy measurement dominates when the uncertainties due to other factors are small enough. In another word, the uncertainty of measured value of the beam energy will determine the final accuracy of resonance parameters.

Table 4: Errors(%) measured/estimated for scan experiment at BESII/BESIII [23].

<table>
<thead>
<tr>
<th>mode</th>
<th>Fitting</th>
<th>Sys.</th>
<th>Lum.</th>
<th>δE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Scan at BESII</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hadron</td>
<td>6.1</td>
<td>2.2</td>
<td>3.2</td>
<td>3.3</td>
</tr>
<tr>
<td>π⁺π⁻J/ψ</td>
<td>6.2</td>
<td>2.2</td>
<td>3.2</td>
<td>3.2</td>
</tr>
<tr>
<td>µ⁺µ⁻</td>
<td>0.5</td>
<td>3.4</td>
<td>3.2</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>Scan at BESIII</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hadron</td>
<td>0.6</td>
<td>&lt; 1.1</td>
<td>&lt; 1</td>
<td>3.3 → 1.6</td>
</tr>
<tr>
<td>π⁺π⁻J/ψ</td>
<td>0.6</td>
<td>&lt; 1.1</td>
<td>&lt; 1</td>
<td>3.2 → 1.5</td>
</tr>
<tr>
<td>µ⁺µ⁻</td>
<td>0.05</td>
<td>&lt; 1.1</td>
<td>&lt; 1</td>
<td>3.5 → 1.7</td>
</tr>
</tbody>
</table>

4 Effect on generic physics analysis

For experiment in $e^+e^-$ collider, special emphasis should be laid on the dependence of the observed cross section on the experimental conditions. One of the most crucial ones is the beam energy setting. As we known, the resonance height is reduced and the position
of its peak is shifted due to the initial state radiative (ISR) correction and the energy spread of the collider. Such effects are peculiarly prominent for the narrow resonances like $J/\psi$ and $\psi(2S)$. Moreover, in $e^+e^-$ experiment, the resonance process is inevitably accompanied by the virtual photon continuum process, but the effects of ISR correction and energy spread on two kinds of processes are rather different [25]. The direct result of such difference will lead to considerable interference effect for some types of processes, for example, the electromagnetic processes [26].

Figure 4: Cross sections in the vicinity of $\psi(2S)$ for inclusive hadrons (a) and $\mu^+\mu^-$ (b) final states. The solid line with arrow indicates the peak position and the dashed line with arrow the position of the other peak. In (b), the dashed line for QED continuum ($\sigma^C$), the dotted line for resonance ($\sigma^R$), the dash-dotted line for interference($\sigma^I$), and the solid line for the total cross section($\sigma^\text{Tot}$).

Figure 4 depicts the observed cross sections of inclusive hadrons (as a representative for hadronic decay) and $\mu$-pairs (as a representative for electromagnetic decay) at $\psi(2S)$ in actual experiments. Two arrows in the figure denote the different positions of the maximum heights of the cross sections. It is obvious that the relative contribution of the resonance and the continuum varies as the energy changes. In actual experiments, data are naturally taken at the energy which yields the maximum inclusive hadronic cross section. This energy does not coincide with the maximum cross section of each exclusive mode. So it is important to know the beam spread and beam energy precisely, which are needed in the delicate task to subtract the contribution from continuum.

For BESII physics analysis, a simple approach once adopted to estimate the uncertainty for inclusive process due to beam energy fluctuation [27], which is at the level of
0.23%. Such an uncertainty is usually negligible for the statistical and systematic uncertainties at the level of 10%. Anyway, for the physics analysis of BESIII, the error may be around 1%-2%, and the uncertainty of 0.23% can not be neglected recklessly. As a matter of fact, for exclusive process, the uncertainty due to beam energy fluctuation can be much greater than that for inclusive process. Taking the analysis $\psi(2S) \rightarrow \tau^+\tau^-$ for example, such an uncertainty can be up to 5% [28].

5 Summary

In this short paper, the significance of energy scale for physics analysis at the BESIII is scrutinized from three aspects: (1) the high accurate measurement of $\tau$ mass, (2) the high accurate cross section scan experiment, and (3) the generic physics analysis at the BESIII with uncertainty at the level of 1%-2%.

For $\tau$ mass measurement, the present study indicates the precision of the beam energy is actually the decisive factor for the accuracy improvement of $\tau$ mass measurement based on large data sample.

For measurement of resonance parameters by scan experiments, the uncertainty due to the beam energy is also a crucial factor for the final results.

The last but not the least is the significance of the accuracy of beam energy for generic physics analysis at the BESIII. Since with large data sample, the uncertainty for a considerable number of channels can reach the level of 1% $\sim$ 2%, under such a case the uncertainty due to the beam energy can hardly be neglected.

In a word, the accurate determination of the beam energy is fairly significant for the physics analyses at the BESIII.

Acknowledgement

We would like to thank Prof. C.Z. Yuan who put forth many suggestions for the improvement of the paper. Thanks also go to Prof. Y.F. Wang, whose suggestion is the start point of this paper.

References


\footnote{For charm and charmonium physics analysis at CLEO-c, the uncertainty already reached the level of 2%-3%. Since the detector of BESIII is similar to that of CLEO-c, but with higher luminosity, smaller uncertainty is expected, which should be at the level of 1%-2%.}


